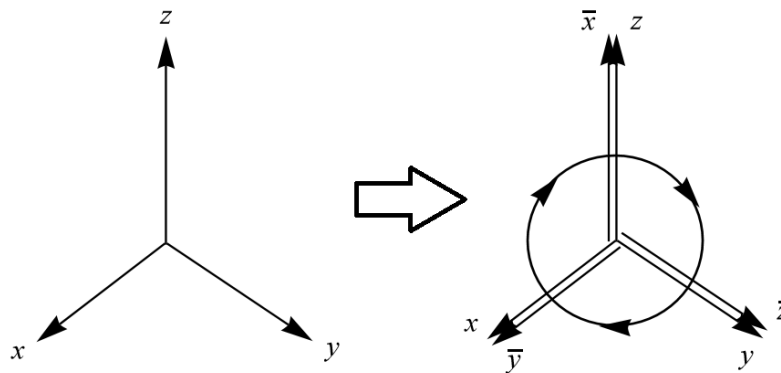


## Problem 1.9

Find the transformation matrix  $R$  that describes a rotation by  $120^\circ$  about an axis from the origin through the point  $(1, 1, 1)$ . The rotation is clockwise as you look down the axis toward the origin.

### Solution

Draw an  $xyz$ -coordinate system, looking at the origin from the point  $(1, 1, 1)$ .



A clockwise rotation of  $120^\circ$  about the line of sight through the origin makes the new  $z$ -axis point in the direction of the  $y$ -axis, makes the new  $y$ -axis point in the direction of the  $x$ -axis, and makes the new  $x$ -axis point in the direction of the  $z$ -axis. As a result, a vector  $\mathbf{A}$  that has the components,  $A_x$ ,  $A_y$ , and  $A_z$ , in the old coordinate system will have the components,

$$\bar{A}_x = A_z$$

$$\bar{A}_y = A_x$$

$$\bar{A}_z = A_y,$$

in the new coordinate system. These three equations can be written compactly as a matrix equation.

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

Therefore, the transformation matrix is

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$