Problem 1.4

Use the cross product to find the components of the unit vector $\hat{\mathbf{n}}$ perpendicular to the shaded plane in Fig. 1.11.

Solution

Fig. 1.11 is shown below.

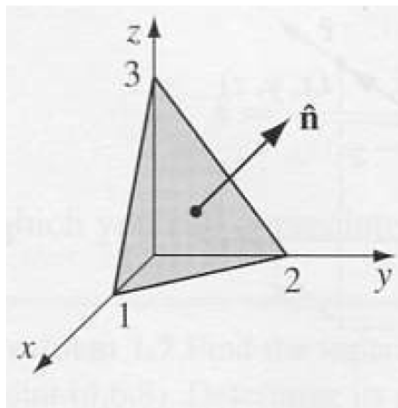

FIGURE 1.11

Let $\mathbf{r}_1$ be the displacement vector from $(1,0,0)$ to $(0,0,3)$ and let $\mathbf{r}_2$ be the displacement vector from $(1,0,0)$ to $(0,2,0)$.

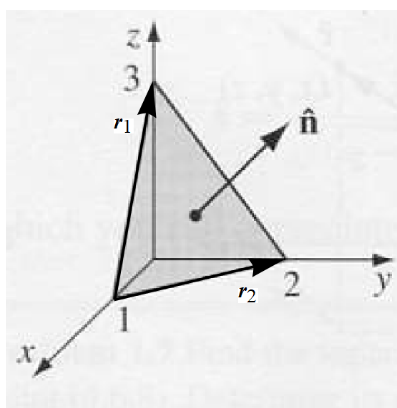

FIGURE 1.11

$$\mathbf{r}_1 = \langle 0,0,3\rangle - \langle 1,0,0\rangle = \langle -1,0,3\rangle$$
$$\mathbf{r}_2 = \langle 0,2,0\rangle - \langle 1,0,0\rangle = \langle -1,2,0\rangle$$

Divide each of them by their respective magnitudes to get the unit vectors.

$$\hat{\mathbf{r}}_1 = \frac{\mathbf{r}_1}{|\mathbf{r}_1|} = \frac{\langle -1,0,3\rangle}{\sqrt{(-1)^2+0^2+3^2}} = \frac{1}{\sqrt{10}}\langle -1,0,3\rangle = \left\langle -\frac{1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}}\right\rangle$$
$$\hat{\mathbf{r}}_2 = \frac{\mathbf{r}_2}{|\mathbf{r}_2|} = \frac{\langle -1,2,0\rangle}{\sqrt{(-1)^2+2^2+0^2}} = \frac{1}{\sqrt{5}}\langle -1,2,0\rangle = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right\rangle$$

The desired unit vector $\hat{\mathbf{n}}$ is obtained by taking the cross product of $\hat{\mathbf{r}}_2$ and $\hat{\mathbf{r}}_1$.

$$\begin{aligned}
\hat{\mathbf{n}} &= \hat{\mathbf{r}}_2 \times \hat{\mathbf{r}}_1 \\
&= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ -\frac{1}{\sqrt{10}} & 0 & \frac{3}{\sqrt{10}} \end{vmatrix} \\
&= \frac{1}{\sqrt{5}} \frac{1}{\sqrt{10}} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} \\
&= \frac{1}{\sqrt{50}}(6\hat{\mathbf{x}} + 3\hat{\mathbf{y}} + 2\hat{\mathbf{z}})
\end{aligned}$$

Therefore,

$$\hat{\mathbf{n}} = \frac{6}{\sqrt{50}}\hat{\mathbf{x}} + \frac{3}{\sqrt{50}}\hat{\mathbf{y}} + \frac{2}{\sqrt{50}}\hat{\mathbf{z}}.$$