

Vector Identity (7)

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

Proof

$$\begin{aligned}
 \nabla \times (f\mathbf{A}) &= \left(\sum_{i=1}^3 \boldsymbol{\delta}_i \frac{\partial}{\partial x_i} \right) \times \left[f \left(\sum_{j=1}^3 \boldsymbol{\delta}_j A_j \right) \right] \\
 &= \left(\sum_{i=1}^3 \boldsymbol{\delta}_i \frac{\partial}{\partial x_i} \right) \times \left(\sum_{j=1}^3 \boldsymbol{\delta}_j A_j f \right) \\
 &= \sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial}{\partial x_i} A_j f \\
 &= \sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \left(\frac{\partial A_j}{\partial x_i} f + A_j \frac{\partial f}{\partial x_i} \right) \\
 &= \sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial A_j}{\partial x_i} f + \sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) A_j \frac{\partial f}{\partial x_i} \\
 &= f \left[\sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial A_j}{\partial x_i} \right] + \sum_{i=1}^3 \sum_{j=1}^3 (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) A_j \frac{\partial f}{\partial x_i} \\
 &= f \left[\left(\sum_{i=1}^3 \boldsymbol{\delta}_i \frac{\partial}{\partial x_i} \right) \times \left(\sum_{j=1}^3 \boldsymbol{\delta}_j A_j \right) \right] + \left(\sum_{i=1}^3 \boldsymbol{\delta}_i \frac{\partial f}{\partial x_i} \right) \times \left(\sum_{j=1}^3 \boldsymbol{\delta}_j A_j \right) \\
 &= f(\nabla \times \mathbf{A}) + (\nabla f) \times \mathbf{A} \\
 &= f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)
 \end{aligned}$$