# Problem A.19

Find the eigenvalues and eigenvectors of the following matrix:

$$\mathsf{M} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Can this matrix be diagonalized?

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**Solution**

The aim here is to solve the eigenvalue problem for the given matrix.

$$\mathsf{Ma} = \lambda \mathsf{a}$$

Bring $\lambda\mathsf{a}$ to the left side and combine the terms.

$$(\mathsf{M} - \lambda \mathsf{I})\mathsf{a} = \mathsf{0} \tag{1}$$

Since $\mathsf{a} \neq \mathsf{0}$, the matrix in parentheses must be singular, that is,

$$\det(\mathsf{M} - \lambda \mathsf{I}) = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 = 0$$

$$\lambda = 1.$$

To find the corresponding eigenvector, plug it back into equation (1).

$$(\mathsf{M} - \lambda \mathsf{I})\mathsf{a} = \mathsf{0}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Turn this matrix equation into a system of equations and solve for either $a_1$ or $a_2$.

$$\left.\begin{aligned} a_2 &= 0 \\ 0 &= 0 \end{aligned}\right\}$$

$$\mathsf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \end{pmatrix}$$

Therefore, the eigenvector corresponding to $\lambda = 1$ is

$$\mathsf{a} = a_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

where $a_1$ is an arbitrary constant (due to the fact that the eigenvalue problem is homogeneous). Since $\mathsf{M}$ is a $2 \times 2$ matrix and there's only one eigenvector, the $\mathsf{S}^{-1}$ matrix can't be constructed, meaning $\mathsf{M}$ can't be diagonalized.