

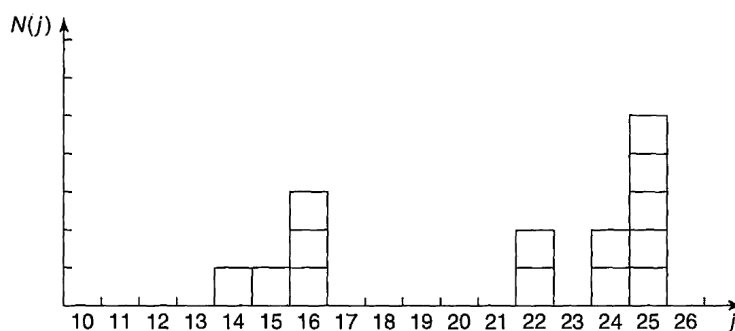
Problem 1.1

For the distribution of ages in the example in Section 1.3.1:

- Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.
- Determine Δj for each j , and use Equation 1.11 to compute the standard deviation.
- Use your results in (a) and (b) to check Equation 1.12.

Solution

The age distribution is shown in Figure 1.5 on page 9.



j represents the age, and $N(j) = N_j$ represents the number of people with age j .

$$N(14) = 1$$

$$N(15) = 1$$

$$N(16) = 3$$

$$N(22) = 2$$

$$N(24) = 2$$

$$N(25) = 5$$

$N_j = 0$ for all other values of j . Now compute the desired quantities, $\langle j \rangle$ and $\langle j^2 \rangle$.

$$\langle j \rangle = \frac{\sum_j j N_j}{\sum_j N_j} = \frac{14 \cdot 1 + 15 \cdot 1 + 16 \cdot 3 + 22 \cdot 2 + 24 \cdot 2 + 25 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = 21$$

$$\langle j^2 \rangle = \frac{\sum_j j^2 N_j}{\sum_j N_j} = \frac{14^2 \cdot 1 + 15^2 \cdot 1 + 16^2 \cdot 3 + 22^2 \cdot 2 + 24^2 \cdot 2 + 25^2 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = \frac{3217}{7}$$

According to Equation 1.12, the formula for the standard deviation is given by

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2} = \sqrt{\frac{3217}{7} - 21^2} = \sqrt{\frac{130}{7}} \approx 4.31. \quad (1.12)$$

Δj represents how far j is from the average: $\Delta j = j - \langle j \rangle = j - 21$.

j	Δj	$(\Delta j)^2$
14	-7	49
15	-6	36
16	-5	25
22	1	1
24	3	9
25	4	16

Use Equation 1.11 to determine the variance.

$$\sigma^2 = \langle (\Delta j)^2 \rangle = \frac{\sum_j (\Delta j)^2 N_j}{\sum_j N_j} = \frac{49 \cdot 1 + 36 \cdot 1 + 25 \cdot 3 + 1 \cdot 2 + 9 \cdot 2 + 16 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = \frac{130}{7} \quad (1.11)$$

Therefore, taking the square root of both sides,

$$\sigma = \sqrt{\frac{130}{7}} \approx 4.31,$$

which is the same as before.