Problem 1.1

For the distribution of ages in the example in Section 1.3.1:

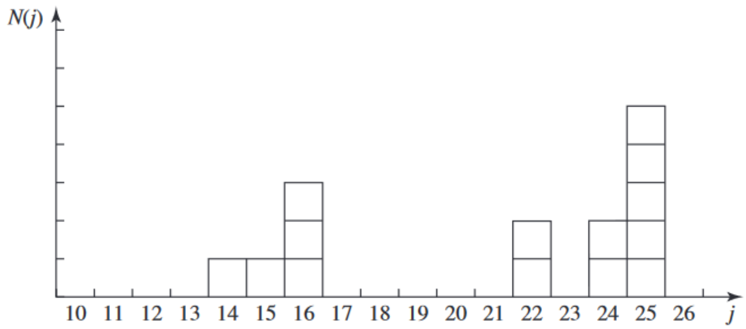
(a) Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.

(b) Determine Δj for each j, and use Equation 1.11 to compute the standard deviation.

(c) Use your results in (a) and (b) to check Equation 1.12.

Solution

The age distribution is shown in Figure 1.5 on page 9.



j represents the age, and $N(j) = N_j$ represents the number of people with age j.

$$\begin{aligned} N(14) &= 1 \\ N(15) &= 1 \\ N(16) &= 3 \\ N(22) &= 2 \\ N(24) &= 2 \\ N(25) &= 5 \end{aligned}$$

$N_j = 0$ for all other values of j. Now compute the desired quantities, $\langle j \rangle$ and $\langle j^2 \rangle$.

$$\langle j \rangle = \frac{\sum_j j N_j}{\sum_j N_j} = \frac{14 \cdot 1 + 15 \cdot 1 + 16 \cdot 3 + 22 \cdot 2 + 24 \cdot 2 + 25 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = 21$$

$$\langle j^2 \rangle = \frac{\sum_j j^2 N_j}{\sum_j N_j} = \frac{14^2 \cdot 1 + 15^2 \cdot 1 + 16^2 \cdot 3 + 22^2 \cdot 2 + 24^2 \cdot 2 + 25^2 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = \frac{3217}{7}$$

According to Equation 1.12, the formula for the standard deviation is given by

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2} = \sqrt{\frac{3217}{7} - 21^2} = \sqrt{\frac{130}{7}} \approx 4.31. \tag{1.12}$$

Δj represents how far j is from the average: $\Delta j = j - \langle j \rangle = j - 21$.

j	Δj	$(\Delta j)^2$
14	-7	49
15	-6	36
16	-5	25
22	1	1
24	3	9
25	4	16

Use Equation 1.11 to determine the variance.

$$\sigma^2 = \langle (\Delta j)^2 \rangle = \frac{\sum_j (\Delta j)^2 N_j}{\sum_j N_j} = \frac{49 \cdot 1 + 36 \cdot 1 + 25 \cdot 3 + 1 \cdot 2 + 9 \cdot 2 + 16 \cdot 5}{1 + 1 + 3 + 2 + 2 + 5} = \frac{130}{7} \tag{1.11}$$

Therefore, taking the square root of both sides,

$$\sigma = \sqrt{\frac{130}{7}} \approx 4.31,$$

which is the same as before.