Problem 1.10

Consider the first 25 digits in the decimal expansion of π $(3, 1, 4, 1, 5, 9, \ldots)$.

(a) If you selected one number at random, from this set, what are the probabilities of getting each of the 10 digits?

(b) What is the most probable digit? What is the median digit? What is the average value?

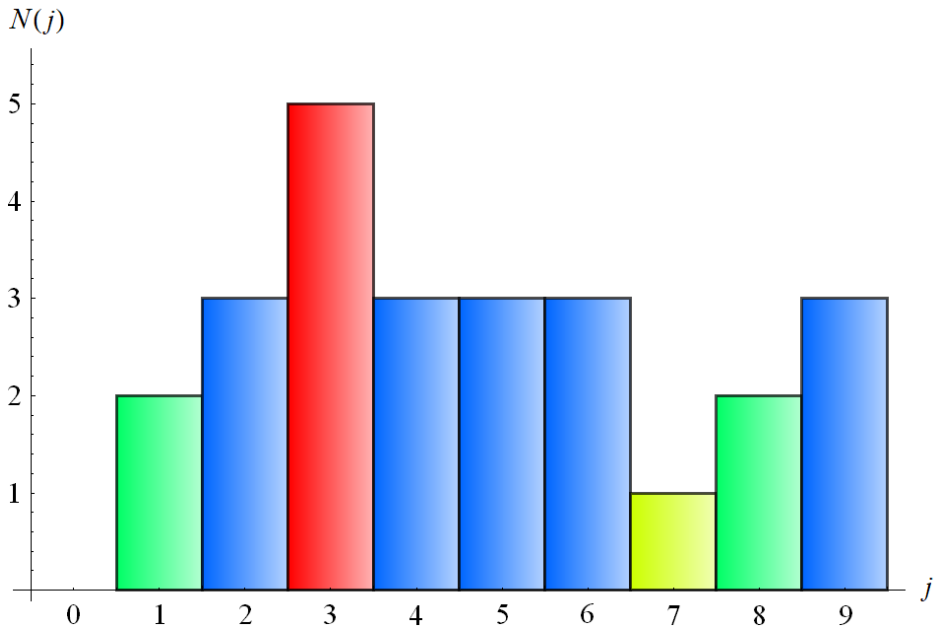
(c) Find the standard deviation for this distribution.

Solution

The decimal expansion of π to the first 25 digits is

$$3.14159265358979323846264 3\ldots.$$

Below is a histogram that shows the frequency of each digit.



The probability of choosing a particular digit is the frequency over the total number of digits.

$$0: \quad \frac{0}{25} = 0.00 \qquad 5: \quad \frac{3}{25} = 0.12$$

$$1: \quad \frac{2}{25} = 0.08 \qquad 6: \quad \frac{3}{25} = 0.12$$

$$2: \quad \frac{3}{25} = 0.12 \qquad 7: \quad \frac{1}{25} = 0.04$$

$$3: \quad \frac{5}{25} = 0.20 \qquad 8: \quad \frac{2}{25} = 0.08$$

$$4: \quad \frac{3}{25} = 0.12 \qquad 9: \quad \frac{3}{25} = 0.12$$

Use the histogram to write the set of all 25 digits from smallest to largest.

$$\{1, 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, \color{magenta}{4}, 5, 5, 5, 6, 6, 6, 7, 8, 8, 9, 9, 9\}$$

The median is 4, the highlighted number in the center of this set (if there were two, their average would be taken); the most probable digit chosen is 3, the number with the highest frequency; the average value is

$$\begin{aligned}\langle j \rangle = \frac{\sum_j jN(j)}{\sum_j N(j)} &= \frac{0(0) + 1(2) + 2(3) + 3(5) + 4(3) + 5(3) + 6(3) + 7(1) + 8(2) + 9(3)}{0 + 2 + 3 + 5 + 3 + 3 + 3 + 1 + 2 + 3} \\ &= \frac{118}{25} \\ &= 4.72;\end{aligned}$$

the average square value is

$$\begin{aligned}\langle j^2 \rangle = \frac{\sum_j j^2N(j)}{\sum_j N(j)} &= \frac{0^2(0) + 1^2(2) + 2^2(3) + 3^2(5) + 4^2(3) + 5^2(3) + 6^2(3) + 7^2(1) + 8^2(2) + 9^2(3)}{0 + 2 + 3 + 5 + 3 + 3 + 3 + 1 + 2 + 3} \\ &= \frac{710}{25} \\ &= 28.4;\end{aligned}$$

and the standard deviation of the distribution is

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2} = \sqrt{28.4 - 4.72^2} \approx 2.47.$$