Problem 1.6

Why can't you do integration-by-parts directly on the middle expression in Equation 1.29—pull the time derivative over onto x, note that $\partial x/\partial t = 0$, and conclude that $d\langle x\rangle/dt = 0$?

Solution

Equation 1.29 is at the bottom of page 16.

$$\frac{d\langle x\rangle}{dt} = \int_{-\infty}^{\infty} x\frac{\partial}{\partial t}|\Psi(x,t)|^2\,dx = \frac{i\hbar}{2m}\int_{-\infty}^{\infty} x\frac{\partial}{\partial x}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \frac{\partial\Psi^*}{\partial x}\Psi\right)dx \tag{1.29}$$

The integral in the middle can't be done by parts because the derivative and integral are taken with respect to different variables, t and x. The point of using the Schrödinger equation (and its complex-conjugated equation) to write the integrand as shown on the right side is so that integration-by-parts can be applied.