# Problem 2.13

A particle in the harmonic oscillator potential starts out in the state

$$\Psi(x,0) = A[3\psi_0(x) + 4\psi_1(x)].$$

**(a)** Find $A$.

**(b)** Construct $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Don't get too excited if $|\Psi(x,t)|^2$ oscillates at exactly the classical frequency; what would it have been had I specified $\psi_2(x)$, instead of $\psi_1(x)$?[31]

**(c)** Find $\langle x\rangle$ and $\langle p\rangle$. Check that Ehrenfest's theorem (Equation 1.38) holds, for this wave function.

**(d)** If you measured the energy of this particle, what values might you get, and with what probabilities?

---

**Solution**

Start by normalizing the initial wave function: Require that the integral of $|\Psi(x,0)|^2$ over the whole line is 1 and solve for $A$. It's important to know that the eigenstates of the harmonic oscillator are orthonormal in order to evaluate the following integrals.

$$\begin{aligned}
1 &= \int_{-\infty}^{\infty} |\Psi(x,0)|^2\,dx \\
&= \int_{-\infty}^{\infty} \Psi(x,0)\Psi^*(x,0)\,dx \\
&= \int_{-\infty}^{\infty} A[3\psi_0(x) + 4\psi_1(x)]A[3\psi_0^*(x) + 4\psi_1^*(x)]\,dx \\
&= A^2\int_{-\infty}^{\infty} [9\psi_0(x)\psi_0^*(x) + 12\psi_0(x)\psi_1^*(x) + 12\psi_1(x)\psi_0^*(x) + 16\psi_1(x)\psi_1^*(x)]\,dx \\
&= A^2\left[9\underbrace{\int_{-\infty}^{\infty}\psi_0(x)\psi_0^*(x)\,dx}_{=1} + 12\underbrace{\int_{-\infty}^{\infty}\psi_0(x)\psi_1^*(x)\,dx}_{=0}\right. \\
&\qquad\qquad \left. + 12\underbrace{\int_{-\infty}^{\infty}\psi_1(x)\psi_0^*(x)\,dx}_{=0} + 16\underbrace{\int_{-\infty}^{\infty}\psi_1(x)\psi_1^*(x)\,dx}_{=1}\right] \\
&= A^2[9(1) + 16(1)] \\
&= 25A^2
\end{aligned}$$

Solve for $A$.

$$A = \frac{1}{5}$$

Therefore, the wave function is initially

$$\Psi(x,0) = \frac{3}{5}\psi_0(x) + \frac{4}{5}\psi_1(x) = \frac{3}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right) + \frac{4}{5}\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4} x\exp\left(-\frac{m\omega}{2\hbar}x^2\right).$$

[31] However, $\langle x\rangle$ *does* oscillate at the classical frequency—see Problem 3.40.

The general solution to the Schrödinger equation with a harmonic oscillator potential was determined in Problem 2.10.

$$\begin{aligned}\Psi(x,t) &= B_0\psi_0(x)\phi_0(t) + B_1\psi_1(x)\phi_1(t) + B_2\psi_2(x)\phi_2(t) + \cdots \\ &= B_0\psi_0(x)e^{-iE_0t/\hbar} + B_1\psi_1(x)e^{-iE_1t/\hbar} + B_2\psi_2(x)e^{-iE_2t/\hbar} + \cdots \\ &= B_0\psi_0(x)e^{-i\omega t/2} + B_1\psi_1(x)e^{-3i\omega t/2} + B_2\psi_2(x)e^{-5i\omega t/2} + \cdots \\ &= B_0\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-i\omega t/2} \\ &\qquad + B_1\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4} x\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-3i\omega t/2} \\ &\qquad\qquad + B_2\left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-5i\omega t/2} + \cdots\end{aligned}$$

Set $t = 0$.

$$\begin{aligned}\Psi(x,0) &= B_0\psi_0(x) + B_1\psi_1(x) + B_2\psi_2(x) + \cdots \\ &= B_0\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\ &\qquad + B_1\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4} x\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\ &\qquad\qquad + B_2\left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right) + \cdots\end{aligned}$$

Matching the coefficients in $\Psi(x,0)$, we obtain

$$B_0 = \frac{3}{5}$$

$$B_1 = \frac{4}{5}$$

$$B_n = 0, \quad n \geq 2,$$

which means the general solution reduces to

$$\Psi(x,t) = \frac{3}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-i\omega t/2} + \frac{4}{5}\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4} x\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-3i\omega t/2}$$

$$\boxed{\Psi(x,t) = \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(3e^{-i\omega t/2} + 4\sqrt{\frac{2m\omega}{\hbar}}xe^{-3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right).}$$

To see what the probabilities of measuring $E_0 = \hbar\omega/2$ and $E_1 = 3\hbar\omega/2$ are, write the wave function in terms of the eigenstates.

$$\Psi(x,t) = \frac{3}{5}\psi_0(x)e^{-iE_0t/\hbar} + \frac{4}{5}\psi_1(x)e^{-iE_1t/\hbar} \quad \Rightarrow \quad \begin{cases} P(E_0) = \left|\dfrac{3}{5}\right|^2 = \dfrac{9}{25} \\ P(E_1) = \left|\dfrac{4}{5}\right|^2 = \dfrac{16}{25}\end{cases}$$

Now that the wave function is known, the probability distribution for the particle's position at time $t$ can be calculated.

$$\begin{aligned}
|\Psi(x,t)|^2 &= \Psi(x,t)\Psi^*(x,t) \\
&= \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(3e^{-i\omega t/2} + 4\sqrt{\frac{2m\omega}{\hbar}}xe^{-3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\
&\qquad \times \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(3e^{i\omega t/2} + 4\sqrt{\frac{2m\omega}{\hbar}}xe^{3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + 16\left(\frac{2m\omega}{\hbar}\right)x^2 + 12\sqrt{\frac{2m\omega}{\hbar}}xe^{i\omega t} + 12\sqrt{\frac{2m\omega}{\hbar}}xe^{-i\omega t}\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + \frac{32m\omega}{\hbar}x^2 + 12\sqrt{\frac{2m\omega}{\hbar}}x(e^{i\omega t} + e^{-i\omega t})\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + \frac{32m\omega}{\hbar}x^2 + 12\sqrt{\frac{2m\omega}{\hbar}}x(2\cos\omega t)\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right)
\end{aligned}$$

$$\boxed{|\Psi(x,t)|^2 = \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left(9 + \frac{32m\omega}{\hbar}x^2 + 24\sqrt{\frac{2m\omega}{\hbar}}x\cos\omega t\right)\exp\left(-\frac{m\omega}{\hbar}x^2\right).}$$

Suppose the provided initial condition were instead

$$\Psi(x,0) = A[3\psi_0(x) + 4\psi_2(x)].$$

The normalization constant $A$ would still be the same: $A = 1/5$.

$$\Psi(x,0) = \frac{3}{5}\psi_0(x) + \frac{4}{5}\psi_2(x)$$

The general solution would be

$$\begin{aligned}
\Psi(x,t) &= \frac{3}{5}\psi_0(x)e^{-iE_0t/\hbar} + \frac{4}{5}\psi_2(x)e^{-iE_2t/\hbar} \\
&= \frac{3}{5}\psi_0(x)e^{-i\omega t/2} + \frac{4}{5}\psi_2(x)e^{-5i\omega t/2} \\
&= \frac{3}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-i\omega t/2} + \frac{4}{5}\left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right)e^{-5i\omega t/2} \\
&= \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left[3e^{-i\omega t/2} + \frac{4}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{-5i\omega t/2}\right]\exp\left(-\frac{m\omega}{2\hbar}x^2\right),
\end{aligned}$$

and the probability distribution for the particle's position at time $t$ would be

$$\begin{aligned}
|\Psi(x,t)|^2 &= \Psi(x,t)\Psi^*(x,t) \\
&= \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left[3e^{-i\omega t/2} + \frac{4}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{-5i\omega t/2}\right]\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\
&\qquad \times \frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left[3e^{i\omega t/2} + \frac{4}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{5i\omega t/2}\right]\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + 8\left(\frac{2m\omega}{\hbar}x^2 - 1\right)^2 + \frac{12}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{2i\omega t} + \frac{12}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{-2i\omega t}\right]e^{-\frac{m\omega}{\hbar}x^2} \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + 8\left(\frac{2m\omega}{\hbar}x^2 - 1\right)^2 + \frac{12}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)(e^{2i\omega t} + e^{-2i\omega t})\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + 8\left(\frac{2m\omega}{\hbar}x^2 - 1\right)^2 + \frac{12}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)(2\cos 2\omega t)\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right) \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9 + 8\left(\frac{2m\omega}{\hbar}x^2 - 1\right)^2 + 12\sqrt{2}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)\cos 2\omega t\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right).
\end{aligned}$$

Getting on with the problem, use the boxed probability distribution to determine the expectation value of $x$ at time $t$.

$$\begin{aligned}
\langle x\rangle &= \int_{-\infty}^{\infty}\Psi^*(x,t)(x)\Psi(x,t)\,dx \\
&= \int_{-\infty}^{\infty} x|\Psi(x,t)|^2\,dx \\
&= \int_{-\infty}^{\infty}\frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}x\left(9 + \frac{32m\omega}{\hbar}x^2 + 24\sqrt{\frac{2m\omega}{\hbar}}x\cos\omega t\right)\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx \\
&= \frac{1}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\left[9\underbrace{\int_{-\infty}^{\infty}x\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx}_{=\,0} + \frac{32m\omega}{\hbar}\underbrace{\int_{-\infty}^{\infty}x^3\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx}_{=\,0}\right. \\
&\qquad\qquad\left. + 24\sqrt{\frac{2m\omega}{\hbar}}\cos\omega t\int_{-\infty}^{\infty}x^2\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx\right] \\
&= \frac{24}{25}\sqrt{\frac{2}{\pi}}\cos\omega t\int_{-\infty}^{\infty}\frac{m\omega}{\hbar}x^2\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx
\end{aligned}$$

Make the following substitution.

$$\xi = \sqrt{\frac{m\omega}{\hbar}}x$$

$$d\xi = \sqrt{\frac{m\omega}{\hbar}}\,dx \quad\rightarrow\quad dx = \sqrt{\frac{\hbar}{m\omega}}\,d\xi$$

Consequently,

$$\begin{aligned}
\langle x\rangle &= \frac{24}{25}\sqrt{\frac{2}{\pi}}\cos\omega t\int_{-\infty}^{\infty}\xi^2 e^{-\xi^2}\left(\sqrt{\frac{\hbar}{m\omega}}\,d\xi\right)\\
&= \frac{24}{25}\sqrt{\frac{2\hbar}{\pi m\omega}}\cos\omega t\int_{-\infty}^{\infty}\xi^2 e^{-\xi^2}\,d\,\xi\\
&= \frac{48}{25}\sqrt{\frac{2\hbar}{\pi m\omega}}\cos\omega t\int_{0}^{\infty}\xi^2 e^{-\xi^2}\,d\,\xi\\
&= \frac{48}{25}\sqrt{\frac{2\hbar}{\pi m\omega}}\cos\omega t\cdot\sqrt{\pi}\,\frac{2!}{1!}\left(\frac{1}{2}\right)^3\\
&= \frac{12}{25}\sqrt{\frac{2\hbar}{m\omega}}\cos\omega t.
\end{aligned}$$

According to Ehrenfest's theorem, the expectation value of $p$ at time $t$ is given by

$$\langle p\rangle = m\langle v\rangle = m\frac{d\langle x\rangle}{dt} = m\frac{d}{dt}\left(\frac{12}{25}\sqrt{\frac{2\hbar}{m\omega}}\cos\omega t\right) = m\left(-\frac{12}{25}\sqrt{\frac{2\hbar}{m\omega}}\omega\sin\omega t\right) = -\frac{12}{25}\sqrt{2\hbar m\omega}\sin\omega t.$$

Check this result by calculating the expectation value of $p$ at time $t$ manually.

$$\begin{aligned}
\langle p\rangle &= \int_{-\infty}^{\infty}\Psi^*(x,t)\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi(x,t)\,dx\\
&= -i\hbar\int_{-\infty}^{\infty}\Psi^*(x,t)\frac{\partial\Psi}{\partial x}\,dx\\
&= -i\hbar\int_{-\infty}^{\infty}\frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(3e^{i\omega t/2}+4\sqrt{\frac{2m\omega}{\hbar}}xe^{3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\\
&\qquad\times\frac{\partial}{\partial x}\frac{1}{5}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(3e^{-i\omega t/2}+4\sqrt{\frac{2m\omega}{\hbar}}xe^{-3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right)dx\\
&= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left(3e^{i\omega t/2}+4\sqrt{\frac{2m\omega}{\hbar}}xe^{3i\omega t/2}\right)\exp\left(-\frac{m\omega}{2\hbar}x^2\right)\\
&\qquad\times\left[\left(4\sqrt{\frac{2m\omega}{\hbar}}e^{-3i\omega t/2}\right)+\left(3e^{-i\omega t/2}+4\sqrt{\frac{2m\omega}{\hbar}}xe^{-3i\omega t/2}\right)\left(-\frac{m\omega}{\hbar}x\right)\right]\exp\left(-\frac{m\omega}{2\hbar}x^2\right)dx\\
&= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left(3e^{i\omega t/2}+4\sqrt{\frac{2m\omega}{\hbar}}xe^{3i\omega t/2}\right)\left[4\sqrt{\frac{2m\omega}{\hbar}}\left(1-\frac{m\omega}{\hbar}x^2\right)e^{-3i\omega t/2}-\frac{3m\omega}{\hbar}xe^{-i\omega t/2}\right]e^{-\frac{m\omega}{\hbar}x^2}\,dx\\
&= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left[-\frac{9m\omega}{\hbar}x+16\left(\frac{2m\omega}{\hbar}\right)x\left(1-\frac{m\omega}{\hbar}x^2\right)+12\sqrt{\frac{2m\omega}{\hbar}}\left(1-\frac{m\omega}{\hbar}x^2\right)e^{-i\omega t}\right.\\
&\qquad\left.-12\sqrt{\frac{2m^3\omega^3}{\hbar^3}}x^2e^{i\omega t}\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx\\
&= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left[\frac{23m\omega}{\hbar}x-\frac{32m^2\omega^2}{\hbar^2}x^3+12\sqrt{\frac{2m\omega}{\hbar}}e^{-i\omega t}-12\sqrt{\frac{2m^3\omega^3}{\hbar^3}}x^2(e^{-i\omega t}+e^{i\omega t})\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx
\end{aligned}$$

Continue simplifying the right side.

$$\begin{aligned}\langle p\rangle &= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left[\frac{23m\omega}{\hbar}x - \frac{32m^2\omega^2}{\hbar^2}x^3 + 12\sqrt{\frac{2m\omega}{\hbar}}e^{-i\omega t} - 12\sqrt{\frac{2m^3\omega^3}{\hbar^3}}x^2(2\cos\omega t)\right]\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx \\
&= -\frac{i\hbar}{25}\sqrt{\frac{m\omega}{\pi\hbar}}\Bigg[\frac{23m\omega}{\hbar}\underbrace{\int_{-\infty}^{\infty} x\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx}_{=0} - \frac{32m^2\omega^2}{\hbar^2}\underbrace{\int_{-\infty}^{\infty} x^3\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx}_{=0} \\
&\qquad + 12\sqrt{\frac{2m\omega}{\hbar}}e^{-i\omega t}\int_{-\infty}^{\infty}\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx - 24\sqrt{\frac{2m^3\omega^3}{\hbar^3}}\cos\omega t\int_{-\infty}^{\infty}x^2\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx\Bigg] \\
&= -\frac{i\hbar}{25}\left[12\frac{m\omega}{\hbar}\sqrt{\frac{2}{\pi}}e^{-i\omega t}\int_{-\infty}^{\infty}\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx - 24\frac{m\omega}{\hbar}\sqrt{\frac{2}{\pi}}\cos\omega t\int_{-\infty}^{\infty}\frac{m\omega}{\hbar}x^2\exp\left(-\frac{m\omega}{\hbar}x^2\right)dx\right]\end{aligned}$$

Make the following substitution.

$$\xi = \sqrt{\frac{m\omega}{\hbar}}x$$

$$d\xi = \sqrt{\frac{m\omega}{\hbar}}\,dx \quad\rightarrow\quad dx = \sqrt{\frac{\hbar}{m\omega}}\,d\xi$$

As a result,

$$\begin{aligned}\langle p\rangle &= -\frac{i\hbar}{25}\left[12\frac{m\omega}{\hbar}\sqrt{\frac{2}{\pi}}e^{-i\omega t}\int_{-\infty}^{\infty}e^{-\xi^2}\left(\sqrt{\frac{\hbar}{m\omega}}\,d\xi\right) - 24\frac{m\omega}{\hbar}\sqrt{\frac{2}{\pi}}\cos\omega t\int_{-\infty}^{\infty}\xi^2 e^{-\xi^2}\left(\sqrt{\frac{\hbar}{m\omega}}\,d\xi\right)\right] \\
&= -\frac{i\hbar}{25}\left(24\sqrt{\frac{2m\omega}{\pi\hbar}}e^{-i\omega t}\int_0^{\infty}e^{-\xi^2}\,d\xi - 48\sqrt{\frac{2m\omega}{\pi\hbar}}\cos\omega t\int_0^{\infty}\xi^2 e^{-\xi^2}\,d\xi\right) \\
&= -\frac{i\hbar}{25}\left[24\sqrt{\frac{2m\omega}{\pi\hbar}}e^{-i\omega t}\cdot\sqrt{\pi}\left(\frac{1}{2}\right) - 48\sqrt{\frac{2m\omega}{\pi\hbar}}\cos\omega t\cdot\sqrt{\pi}\,\frac{2!}{1!}\left(\frac{1}{2}\right)^3\right] \\
&= -\frac{i\hbar}{25}\left(12\sqrt{\frac{2m\omega}{\hbar}}e^{-i\omega t} - 12\sqrt{\frac{2m\omega}{\hbar}}\cos\omega t\right) \\
&= -\frac{12}{25}\sqrt{2\hbar m\omega}\,i(e^{-i\omega t} - \cos\omega t) \\
&= -\frac{12}{25}\sqrt{2\hbar m\omega}\,i[(\cancel{\cos\omega t} - i\sin\omega t) - \cancel{\cos\omega t}] \\
&= -\frac{12}{25}\sqrt{2\hbar m\omega}\sin\omega t,\end{aligned}$$

which confirms one part of Ehrenfest's theorem. Check the other part now.

$$\frac{d\langle p\rangle}{dt} \stackrel{?}{=} \left\langle -\frac{\partial V}{\partial x}\right\rangle$$

$$\frac{d}{dt}\left(-\frac{12}{25}\sqrt{2\hbar m\omega}\sin\omega t\right) \stackrel{?}{=} \left\langle -\frac{\partial}{\partial x}\left(\frac{1}{2}m\omega^2x^2\right)\right\rangle$$

$$-\frac{12}{25}\sqrt{2\hbar m\omega}\,\omega\cos\omega t = \langle -m\omega^2x\rangle = -m\omega^2\langle x\rangle = -m\omega^2\left(\frac{12}{25}\sqrt{\frac{2\hbar}{m\omega}}\cos\omega t\right) = -\frac{12}{25}\sqrt{2\hbar m\omega}\,\omega\cos\omega t$$