Problem 2.17

Show that $\left[Ae^{ikx} + Be^{-ikx}\right]$ and $[C\cos kx + D\sin kx]$ are equivalent ways of writing the same function of x, and determine the constants C and D in terms of A and B, and vice versa. *Comment:* In quantum mechanics, when $V = 0$, the exponentials represent *traveling* waves, and are most convenient in discussing the free particle, whereas sines and cosines correspond to *standing* waves, which arise naturally in the case of the infinite square well.

Solution

Use Euler's formula.

$$\begin{aligned}
Ae^{ikx} + Be^{-ikx} &= Ae^{i(kx)} + Be^{i(-kx)} \\
&= A[\cos(kx) + i\sin(kx)] + B[\cos(-kx) + i\sin(-kx)] \\
&= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx) \\
&= A\cos kx + iA\sin kx + B\cos kx - iB\sin kx \\
&= (A + B)\cos kx + i(A - B)\sin kx
\end{aligned}$$

Use new arbitrary constants, C and D, for $A + B$ and $i(A - B)$, respectively.

$$\begin{aligned}
C &= A + B \\
D &= i(A - B)
\end{aligned}$$

Solve this system of equations for A and B.

$$\begin{aligned}
A &= \frac{1}{2}(C - iD) \\
B &= \frac{1}{2}(C + iD)
\end{aligned}$$