Problem 2.56

Find the first three excited state energies (to five significant digits) for the harmonic oscillator, by wagging the dog (Problem 2.55). For the first (and third) excited state you will need to set **u[0] == 0, u′[0] == 1**.)

TYPO: There's an extra parenthesis at the end of the sentence.

Solution

The dimensionless form of the TISE for the harmonic oscillator is in Equation 2.73 on page 48.

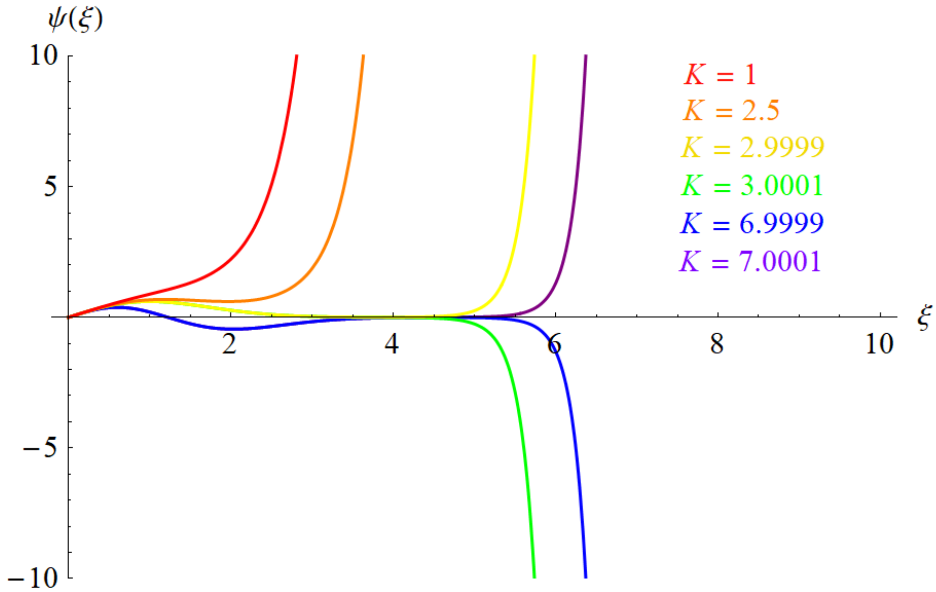
$$\frac{d^2\psi}{d\xi^2} = \left(\xi^2 - K\right)\psi$$

$\xi = \sqrt{m\omega/\hbar}\,x$ is the dimensionless position, and $K = 2E/(\hbar\omega)$ is the dimensionless energy. On the interval $-\infty < \xi < \infty$, the eigenstates $\psi_n(\xi)$ are known to be odd for odd n and even for even n. When evaluating this ODE numerically using the code in Problem 2.55, it's not possible to use boundary conditions at $\pm\infty$. To find the eigenvalues associated with the odd eigenstates, then, one should use boundary conditions for a typical odd function, $\psi(0) = 0$ and $\psi'(0) = 1$. For the eigenvalues associated with the even eigenstates, one should use boundary conditions for a typical even function, $\psi(0) = 1$ and $\psi'(0) = 0$. Note that although $\psi(0) = 0$ and $\psi'(0) = 0$ are boundary conditions for an even function, it will result in the trivial solution.

```
Manipulate[
  Plot[
    Evaluate[
      u[x]/.
        NDSolve[
          {u''[x] - (x^2 - L)*u[x] == 0, u[0] == 0, u'[0] == 1}, u[x], {x, 0, 10}
        ]
      ],
    {x, 0, 10}, PlotRange -> {-10, 10}
  ], {L, 0, 10}
]
```

This code can be used in Mathematica to quickly skim through the values of K and see where the graph flips over.

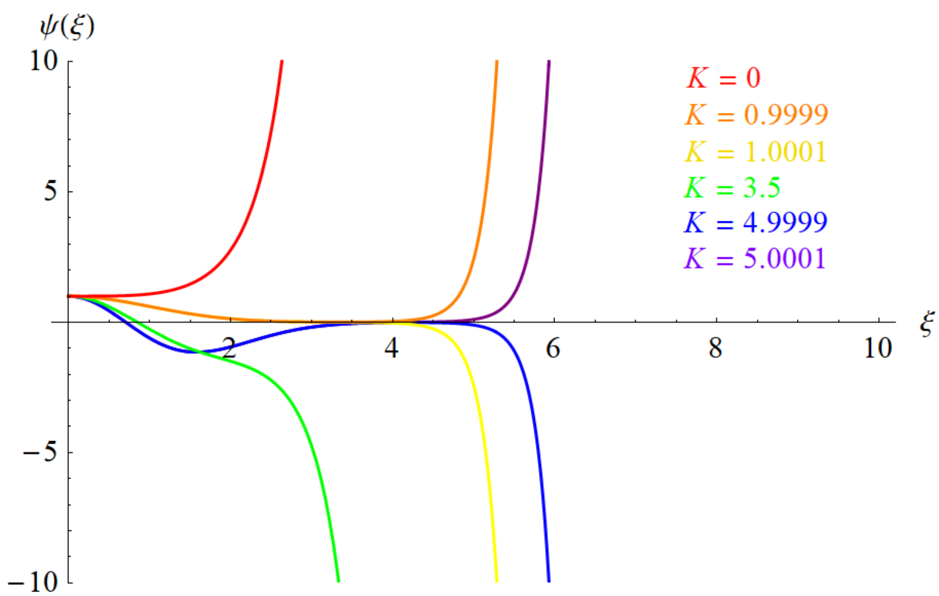
Below is a plot of the numerical solution to the dimensionless TISE with the boundary conditions, $\psi(0) = 0$ and $\psi'(0) = 1$, for several values of K.



The graph of ψ flips over twice in the $0 < K < 10$ interval, once as it passes $K \approx 3.0000$ and once as it passes $K \approx 7.0000$. Therefore, the energies corresponding to the first two odd eigenstates, ψ_1 and ψ_3, are

$$K = \frac{2E}{\hbar\omega} \approx 3.0000 \qquad \Rightarrow \qquad E_1 \approx 1.5000\hbar\omega$$
$$K = \frac{2E}{\hbar\omega} \approx 7.0000 \qquad \Rightarrow \qquad E_3 \approx 3.5000\hbar\omega.$$

Below is a plot of the numerical solution to the dimensionless TISE with the boundary conditions, $\psi(0) = 1$ and $\psi'(0) = 0$, for several values of K.



The graph of ψ flips over twice in the $0 < K < 6$ interval, once as it passes $K \approx 1.0000$ and once as it passes $K \approx 5.0000$. Therefore, the energies corresponding to the first two even eigenstates, ψ_0 and ψ_2, are

$$K = \frac{2E}{\hbar\omega} \approx 1.0000 \qquad \Rightarrow \qquad E_0 \approx 0.50000\hbar\omega$$
$$K = \frac{2E}{\hbar\omega} \approx 5.0000 \qquad \Rightarrow \qquad E_2 \approx 2.5000\hbar\omega.$$