Problem 4.28

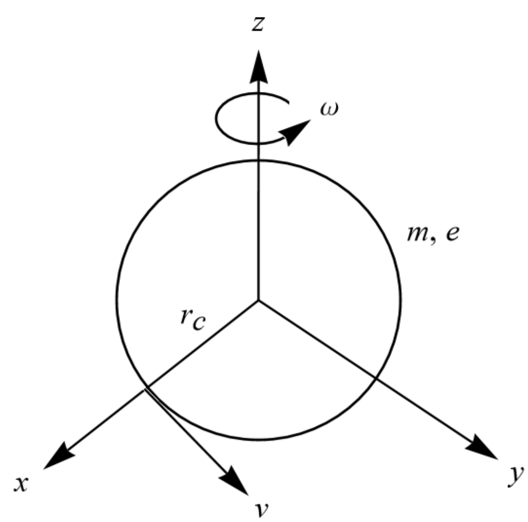
If the electron were a classical solid sphere, with radius

$$r_c = \frac{e^2}{4\pi\epsilon_0 mc^2}, \tag{4.138}$$

(the so-called **classical electron radius**, obtained by assuming the electron's mass is attributable to energy stored in its electric field, via the Einstein formula $E = mc^2$), and its angular momentum is $(1/2)\hbar$, then how fast (in m/s) would a point on the "equator" be moving? Does this model make sense? (Actually, the radius of the electron is known experimentally to be much less than r_c, but this only makes matters worse.)[39]

Solution

Draw a schematic of the spinning electron, assuming it's a solid ball with radius r_c, mass m, and charge e.



The relationship between the angular momentum and the angular speed of a rigid body that's spinning about its axis of symmetry is

$$\mathbf{L} = I\boldsymbol{\omega}.$$

The electron is spinning about the z-axis specifically.

$$L_z = I\omega_z$$

Substitute $\hbar/2$ for L_z and substitute $(2/5)mr_c^2$, the momentum of inertia for a solid sphere spinning about its axis of symmetry, for I.

$$\frac{\hbar}{2} = \left(\frac{2}{5}mr_c^2\right)\omega_z$$

Express the angular speed in terms of v, the linear speed of a point on the equator.

$$\frac{\hbar}{2} = \left(\frac{2}{5}mr_c^2\right)\left(\frac{v}{r_c}\right)$$

[39]If it comforts you to picture the electron as a tiny spinning sphere, go ahead; I do, and I don't think it hurts, as long as you don't take it literally.

Solve for v.

$$\begin{aligned} v &= \frac{5\hbar}{4mr_c} \\ &= \frac{5\hbar}{4m\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)} \\ &= \frac{5\pi\epsilon_0\hbar c^2}{e^2} \\ &\approx \frac{5\pi\left(8.854187817\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(1.054571726\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(299\,792\,458\ \frac{\text{m}}{\text{s}}\right)^2}{(1.602176565\times10^{-19}\ \text{C})^2} \\ &\approx 5.13529\times10^{10}\ \frac{\text{m}}{\text{s}} \end{aligned}$$

This linear speed is

$$\frac{5.13529\times10^{10}}{299\,792\,458}\approx 171.295$$

times the speed of light, which is ridiculous. Apparently, the spin from classical mechanics (due to motion about the center of mass) is not the same as the spin from quantum mechanics (due to intrinsic angular momentum), and it was unwise to set $L_z = \hbar/2$.