# Exercise 1.2.1

Briefly explain the minus sign:

(a) in conservation law (1.2.3) or (1.2.5) if $Q = 0$

(b) in Fourier's law (1.2.8)

(c) in conservation law (1.2.12), $\frac{\partial u}{\partial t} = -\frac{\partial \phi}{\partial x}$

(d) in Fick's law (1.2.13)

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**Solution**

## Part (a)

If $Q = 0$, (1.2.3) and (1.2.5) become

$$\frac{\partial e}{\partial t} = -\frac{\partial \phi}{\partial x}.$$

This equation is a mathematical statement of the first law of thermodynamics, namely that thermal energy is conserved. This relationship must hold for any differential element of a one-dimensional rod. If more thermal energy is flowing out of the right end of an element than is flowing into it from the left end (that is, $\partial \phi/\partial x > 0$), then the thermal energy density of the element will decrease in time ($\partial e/\partial t < 0$). Conversely, if more thermal energy is flowing into the element from the left end than is flowing out of it from the right end (that is, $\partial \phi/\partial x < 0$), then the thermal energy density of the element will increase in time ($\partial e/\partial t > 0$).

## Part (b)

Fourier's law is

$$\phi = -K_0 \frac{\partial u}{\partial x}.$$

It is an empirical relationship that states the heat flux is proportional to the temperature gradient. The minus sign in the equation comes from the fact that thermal energy flows from hotter areas to cooler areas. For example, if the temperature in a one-dimensional rod increases uniformly from left to right (that is, $\partial u/\partial x > 0$), then thermal energy will flow from right to left ($\phi < 0$) and vice-versa.

## Part (c)

The conservation law (1.2.12) is

$$\frac{\partial u}{\partial t} = -\frac{\partial \phi}{\partial x}.$$

It is a mathematical statement of the conservation of mass. This relationship must hold for any differential element in a one-dimensional region. If more mass is flowing out of the right end of an element than is flowing into it from the left end (that is, $\partial \phi/\partial x > 0$), then the mass density of the element will decrease in time ($\partial u/\partial t < 0$). Conversely, if more mass is flowing into the element from the left end than is flowing out of it from the right end (that is, $\partial \phi/\partial x < 0$), then the mass density of the element will increase in time ($\partial u/\partial t > 0$).

**Part (d)**

Fick's law is

$$\phi = -k\frac{\partial u}{\partial x}.$$

It is an empirical relationship that states the mass flux is proportional to the concentration gradient. The minus sign in the equation comes from the fact that mass flows down its concentration gradient. For example, if the concentration of a pollutant in a one-dimensional region increases uniformly from left to right (that is, $\partial u/\partial x > 0$), then mass will flow from right to left ($\phi < 0$) and vice-versa.