Problem 12-10

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

Solution

The acceleration $a = a(t)$ is related to velocity $v = v(t)$ by

$$a = \frac{dv}{dt}.$$

Here, though, the velocity is a function of position, not time: $v = v(s)$. Use the chain rule to evaluate the derivative.

$$\begin{aligned} a &= \frac{dv(s)}{dt} \\ &= \frac{dv}{ds}\frac{ds}{dt} \\ &= \frac{dv}{ds}v \end{aligned}$$

Separate variables.

$$v\,dv = a(s)\,ds$$

Integrate both sides, using Simpson's rule on the right side with $n = 10$.

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a(s)\,ds$$

$$\int_0^v v'\,dv' = \int_1^2 a(s)\,ds$$

$$\frac{v^2}{2} \approx \frac{\frac{2-1}{10}}{3}[a(1) + 4a(1.1) + 2a(1.2) + 4a(1.3) + 2a(1.4) + 4a(1.5) + 2a(1.6) + 4a(1.7) + 2a(1.8) + 4a(1.9) + a(2.0)]$$

$$\frac{v^2}{2} \approx \frac{1}{30}\left[\frac{5}{4} + 4(1.145) + 2(1.049) + 4(0.961) + 2(0.881) + 4(0.808) + 2(0.741) + 4(0.680) + 2(0.625) + 4(0.575) + (0.530)\right]$$

$$\frac{v^2}{2} \approx 0.835$$

Solve for v.

$$v \approx 1.29\ \frac{\text{m}}{\text{s}}$$