Problem 12-14

The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ ft, where t is in seconds. Determine the total distance traveled when $t = 10$ s. What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

Solution

The average velocity is

$$v_{\text{avg}} = \frac{s(10) - s(0)}{(10) - (0)} = \frac{(257) - (7)}{10} = 25\ \frac{\text{ft}}{\text{s}}.$$

Differentiate the given position function to get the velocity.

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{d}{dt}(t^3 - 6t^2 - 15t + 7) \\ &= 3t^2 - 12t - 15 \end{aligned}$$

The total distance travelled in the first 10 seconds is the integral of the speed from $t = 0$ to $t = 10$.

$$\begin{aligned} s_T = \int_0^{10} |v(t)|\, dt &= \int_0^{10} |3t^2 - 12t - 15|\, dt \\ &= \int_0^5 (-3t^2 + 12t + 15)\, dt + \int_5^{10} (3t^2 - 12t - 15)\, dt \\ &= \left(-t^3 + 6t^2 + 15t\right)\Big|_0^5 + \left(t^3 - 6t^2 - 15t\right)\Big|_5^{10} \\ &= [-5^3 + 6(5)^2 + 15(5)] + [10^3 - 6(10)^2 - 15(10) - 5^3 + 6(5)^2 + 15(5)] \\ &= (100) + (350) \\ &= 450 \text{ ft} \end{aligned}$$

Now the average speed can be calculated.

$$\frac{s_T}{\Delta t} = \frac{450 \text{ ft}}{10 \text{ s}} = 45\ \frac{\text{ft}}{\text{s}}$$

The velocity at 10 seconds is

$$v(10) = 3(10)^2 - 12(10) - 15 = 165\ \frac{\text{ft}}{\text{s}}.$$

Differentiate the velocity to get the acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 12t - 15) = 6t - 12$$

Therefore, the acceleration at 10 seconds is

$$a(10) = 6(10) - 12 = 48\ \frac{\text{ft}}{\text{s}}.$$