Problem 12-16

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

Solution

The acceleration and velocity are related by

$$a = \frac{dv}{dt} = -1.5v^{1/2}.$$

Divide both sides by $v^{1/2}$.

$$\frac{\frac{dv}{dt}}{v^{1/2}} = -1.5$$

Rewrite the left side.

$$\frac{d}{dt}(2v^{1/2}) = -1.5$$

Integrate both sides with respect to t.

$$2v^{1/2} = -1.5t + C_1 \tag{1}$$

Use the fact that the initial velocity is 6 m/s to determine C_1.

$$2(6)^{1/2} = -1.5(0) + C_1 \quad \rightarrow \quad C_1 = 2\sqrt{6}$$

As a result, equation (1) becomes

$$2v^{1/2} = -1.5t + 2\sqrt{6}.$$

Solve for v.

$$v(t) = (-0.75t + \sqrt{6})^2$$

Find out how long it takes for the particle to stop: Set $v(t) = 0$ and solve for t.

$$-0.75t + \sqrt{6} = 0 \quad \rightarrow \quad t = \frac{\sqrt{6}}{0.75} = \frac{4}{3}\sqrt{6} \approx 3.27 \text{ s}$$

To find how far the particle moves before coming to a stop, integrate the speed from $t = 0$ to $t = 4\sqrt{6}/3$.

$$\begin{aligned} s_T = \int_0^{\frac{4\sqrt{6}}{3}} |v(t)|\,dt = \int_0^{\frac{4\sqrt{6}}{3}} (-0.75t + \sqrt{6})^2\,dt &= \int_0^{\frac{4\sqrt{6}}{3}} \left(\frac{9}{16}t^2 - 1.5\sqrt{6}t + 6\right) dt \\ &= \left(\frac{3}{16}t^3 - 0.75\sqrt{6}t^2 + 6t\right)\Bigg|_0^{\frac{4\sqrt{6}}{3}} \\ &= \frac{3}{16}\left(\frac{4\sqrt{6}}{3}\right)^3 - 0.75\sqrt{6}\left(\frac{4\sqrt{6}}{3}\right)^2 + 6\left(\frac{4\sqrt{6}}{3}\right) \\ &= 8\sqrt{\frac{2}{3}} \text{ m} \approx 6.53 \text{ m} \end{aligned}$$