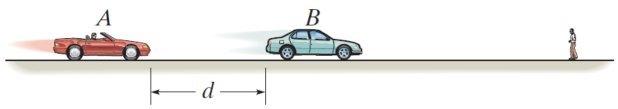
Problem 12-17

Car B is traveling a distance d ahead of car A. Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s^2. It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



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[TYPO: "Between" is one word.]

Solution

Because the acceleration for both cars is constant, the familiar formulas of kinematics apply, in particular,

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Use this formula to model the position (in feet) of both cars, assuming that car B starts to decelerate at $x = 0$ and $t = 0$.

$$\text{Car } B: \quad x_B = 0 + 60t + \frac{1}{2}(-12)t^2, \quad t \geq 0$$

$$\text{Car } A: \quad x_A = \begin{cases} -d + 60t + \frac{1}{2}(0)t^2 & \text{if } 0 \leq t \leq 0.75 \\ (-d + 60 \cdot 0.75) + 60(t - 0.75) + \frac{1}{2}(-15)(t - 0.75)^2 & \text{if } t > 0.75 \end{cases}$$

A car collision occurs if $x_A = x_B$. For $0 \leq t \leq 0.75$, this happens when

$$-d + 60t = 60t + \frac{1}{2}(-12)t^2 \quad \rightarrow \quad t = \sqrt{\frac{d}{6}},$$

and for $t > 0.75$, this happens when

$$(-d + 60 \cdot 0.75) + 60(t - 0.75) + \frac{1}{2}(-15)(t - 0.75)^2 = 60t + \frac{1}{2}(-12)t^2 \quad \rightarrow \quad t = \frac{45 \pm 2\sqrt{405 - 24d}}{12}.$$

The curves for position are parabolas for $t > 0.75$, so they intersect twice at the values of t above. For there to be no intersections, it's necessary that

$$405 - 24d < 0$$

$$d > \frac{405}{24} \text{ ft} \approx 16.9 \text{ ft}.$$