Problem 12-23

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)$ m/s^2, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

Solution

The acceleration and velocity are related by

$$a = \frac{dv}{dt} = -2v.$$

Divide both sides by v.

$$\frac{\frac{dv}{dt}}{v} = -2$$

Rewrite the left side.

$$\frac{d}{dt}\ln|v| = -2$$

The absolute value sign is included since the logarithm's argument can't be negative. Integrate both sides with respect to t.

$$\ln|v| = -2t + C$$

Exponentiate both sides.

$$\begin{aligned}|v| &= e^{-2t+C} \\ &= e^C e^{-2t}\end{aligned}$$

Place $\pm$ on the right side to remove the absolute value sign.

$$v(t) = \pm e^C e^{-2t}$$

Use a new constant A for e^{-2t}.

$$v(t) = Ae^{-2t}$$

Use the fact that $v = 20$ at $t = 0$.

$$v(0) = A = 20$$

The particle's velocity (in meters per second) is then

$$\boxed{v(t) = 20e^{-2t}.}$$

Differentiate it to get the acceleration (in meters per second^2).

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(20e^{-2t})$$

$$\boxed{a(t) = -40e^{-2t}}$$

The velocity and position are related by

$$v = \frac{ds}{dt} = 20e^{-2t}.$$

Integrate both sides with respect to t to get $s(t)$.

$$s(t) = \int (20e^{-2t})\, dt = -10e^{-2t} + D$$

Use the fact that $s = 0$ at $t = 0$ to determine D.

$$s(0) = -10 + D = 0 \quad \rightarrow \quad D = 10$$

The particle's position (in meters) is then

$$\boxed{s(t) = -10e^{-2t} + 10.}$$