Problem 12-26

The acceleration of a particle along a straight line is defined by $a = (2t - 9)$ m/s^2, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

Solution

The acceleration and velocity are related by

$$a = \frac{dv}{dt} = 2t - 9.$$

Integrate both sides with respect to t to get the velocity.

$$\begin{aligned} v(t) &= \int (2t - 9)\, dt \\ &= t^2 - 9t + C_1 \end{aligned}$$

Use the fact that $v = 10$ when $t = 0$ to determine C_1.

$$10 = 0^2 - 9(0) + C_1 \quad \rightarrow \quad C_1 = 10$$

As a result, the velocity (in meters per second) is

$$v(t) = t^2 - 9t + 10.$$

The velocity and position are related by

$$v = \frac{ds}{dt} = t^2 - 9t + 10.$$

Integrate both sides with respect to t to get the position.

$$\begin{aligned} s(t) &= \int (t^2 - 9t + 10)\, dt \\ &= \frac{t^3}{3} - \frac{9}{2}t^2 + 10t + C_2 \end{aligned}$$

Use the fact that $s = 1$ when $t = 0$ to determine C_2.

$$1 = \frac{0^3}{3} - \frac{9}{2}(0)^2 + 10(0) + C_2 \quad \rightarrow \quad C_2 = 1$$

As a result, the position (in meters) is

$$s(t) = \frac{t^3}{3} - \frac{9}{2}t^2 + 10t + 1.$$

Therefore, at $t = 9$ s,

$$s(9) = -30.5 \text{ m}$$

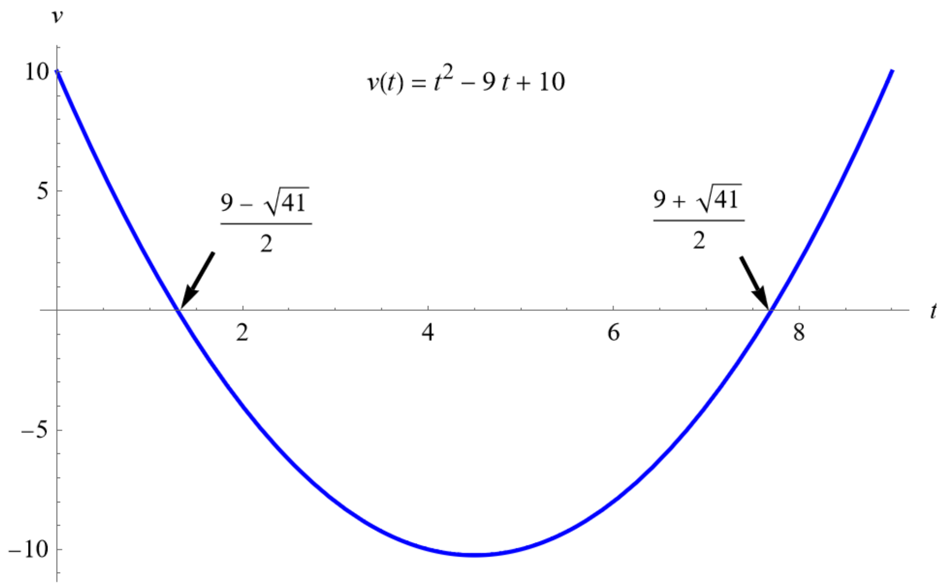
$$v(9) = 10 \ \frac{\text{m}}{\text{s}}.$$

To get the total distance travelled after 9 seconds, integrate the speed from $t = 0$ to $t = 9$.

$$s_{\text{total}} = \int_0^9 |v(t)|\,dt$$

$$= \int_0^9 |t^2 - 9t + 10|\,dt$$

Below is a plot of the velocity versus time.



Find where the velocity is zero.

$$t^2 - 9t + 10 = 0$$

$$t = \frac{9 \pm \sqrt{81 - 4(1)(10)}}{2}$$

$$t = \frac{9 \pm \sqrt{41}}{2}$$

Therefore,

$$s_{\text{total}} = \int_0^{\frac{9-\sqrt{41}}{2}} (t^2 - 9t + 10)\,dt + \int_{\frac{9-\sqrt{41}}{2}}^{\frac{9+\sqrt{41}}{2}} (-t^2 + 9t - 10)\,dt + \int_{\frac{9+\sqrt{41}}{2}}^{9} (t^2 - 9t + 10)\,dt$$

$$= \left(\frac{t^3}{3} - \frac{9}{2}t^2 + 10t\right)\Bigg|_0^{\frac{9-\sqrt{41}}{2}} + \left(-\frac{t^3}{3} + \frac{9}{2}t^2 - 10t\right)\Bigg|_{\frac{9-\sqrt{41}}{2}}^{\frac{9+\sqrt{41}}{2}} + \left(\frac{t^3}{3} - \frac{9}{2}t^2 + 10t\right)\Bigg|_{\frac{9+\sqrt{41}}{2}}^{9}$$

$$= \left(-\frac{63}{4} + \frac{41\sqrt{41}}{12}\right) + \left(\frac{41\sqrt{41}}{6}\right) + \left(-\frac{63}{4} + \frac{41\sqrt{41}}{12}\right)$$

$$= \frac{82\sqrt{41} - 189}{6} \approx 56.0 \text{ m.}$$