Problem 12-3

A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

Solution

The acceleration is obtained by differentiating the given velocity function.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(12 - 3t^2) \\ &= -6t. \end{aligned}$$

Therefore, at $t = 4$ s, the particle's acceleration is

$$a(4) = -24\ \frac{\text{m}}{\text{s}^2}.$$

On the other hand, the position and velocity are related by

$$v = \frac{ds}{dt} = 12 - 3t^2.$$

Integrate both sides with respect to time from $t = 0$ to $t = 10$ to obtain the displacement during this interval.

$$\begin{aligned} \int_0^{10} \frac{ds}{dt}\,dt &= \int_0^{10} (12 - 3t^2)\,dt \\ s(10) - s(0) &= (12t - t^3)\Big|_0^{10} \\ \Delta s &= 12(10) - 10^3 \\ &= -880 \text{ m} \end{aligned}$$

To obtain the total distance the particle travels from $t = 0$ to $t = 10$, integrate the speed over this interval.

$$\begin{aligned} s_T = \int_0^{10} |v(t)|\,dt &= \int_0^{10} |12 - 3t^2|\,dt \\ &= \int_0^{2} (12 - 3t^2)\,dt + \int_2^{10} (3t^2 - 12)\,dt \\ &= (12t - t^3)\Big|_0^{2} + (t^3 - 12t)\Big|_2^{10} \\ &= [12(2) - 2^3] + [10^3 - 12(10) - 2^3 + 12(2)] \\ &= 912 \text{ m} \end{aligned}$$