Problem 12-5

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

Solution

Differentiate the given velocity function to obtain the acceleration.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(6t - 3t^2) \\ &= 6 - 6t \end{aligned}$$

Therefore, the particle's acceleration at $t = 3$ s is

$$a(3) = 6 - 6(3) = -12 \ \frac{\text{m}}{\text{s}^2}.$$

On the other hand, the position is related to velocity by

$$v = \frac{ds}{dt} = 6t - 3t^2.$$

Integrate both sides with respect to t to get $s(t)$.

$$\begin{aligned} s(t) &= \int (6t - 3t^2)\, dt \\ &= 3t^2 - t^3 + C_1 \end{aligned}$$

Use the fact that the initial position is zero to determine C_1.

$$s(0) = C_1 = 0$$

The particle's position (in meters) is then

$$s(t) = 3t^2 - t^3.$$

Therefore, the particle's position at $t = 3$ s is

$$s(3) = 3(3)^2 - (3)^3 = 0.$$

To get the total distance travelled from $t = 0$ s to $t = 3$ s, integrate the speed from 0 to 3.

$$\begin{aligned} s_T = \int_0^3 |v(t)|\, dt = \int_0^3 |6t - 3t^2|\, dt &= \int_0^2 (6t - 3t^2)\, dt + \int_2^3 (3t^2 - 6t)\, dt \\ &= (3t^2 - t^3)\Big|_0^2 + (t^3 - 3t^2)\Big|_2^3 \\ &= [3(2)^2 - 2^3] + [3^3 - 3(3)^2 - 2^3 + 3(2)^2] = 8 \text{ m} \end{aligned}$$

The average speed is

$$\frac{s_T}{\Delta t} = \frac{8 \text{ m}}{3 \text{ s} - 0 \text{ s}} = \frac{8}{3} \ \frac{\text{m}}{\text{s}} \approx 2.67 \ \frac{\text{m}}{\text{s}}.$$