Problem 1-15

Using the SI system of units, show that Eq. 1–2 is a dimensionally homogeneous equation which gives F in newtons. Determine to three significant figures the gravitational force acting between two spheres that are touching each other. The mass of each sphere is 200 kg and the radius is 300 mm.

Solution

Eq. 1–2 is Newton's law of gravitation,

$$F = G\frac{m_1 m_2}{r^2}, \tag{1–2}$$

where G is the universal constant of gravitation:

$$G = 6.67 \times 10^{-11}\ \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}.$$

Check to see if the equation is dimensionally homogeneous.

$$[F] \stackrel{?}{=} \left[G\frac{m_1 m_2}{r^2}\right]$$

$$[\text{N}] \stackrel{?}{=} \left[\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \cdot \frac{\text{kg} \cdot \text{kg}}{\text{m}^2}\right]$$

$$[\text{N}] = \left[\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right]$$

Now use Eq. 1–2 to determine the gravitational force between two touching spheres of mass 200 kg and radius 300 mm. Both spheres can be treated as particles at their respective centers, so the distance for r is $2 \times 300\ \text{mm} = 600\ \text{mm} = 0.6\ \text{m}$.

$$F = G\frac{m_1 m_2}{r^2} = \left(6.67 \times 10^{-11}\ \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}\right)\frac{(200\ \text{kg})(200\ \text{kg})}{(0.6\ \text{m})^2} \approx 7.41 \times 10^{-6}\ \text{N}$$