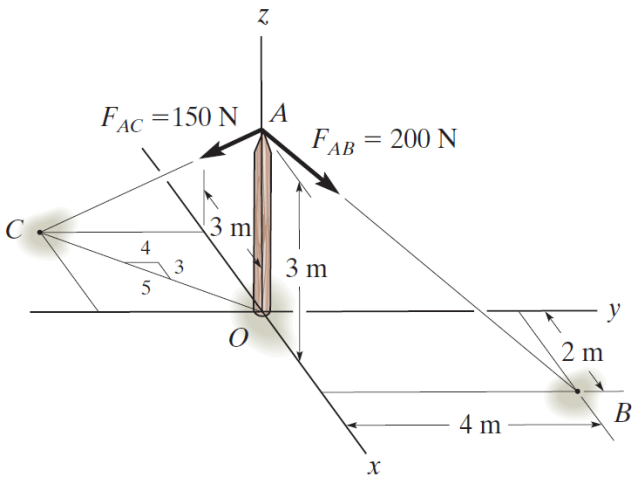
Problem 2-100

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



Prob. 2–100

Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle 0, 0, 3 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 2, 4, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -3, -4, 0 \rangle \text{ m}$$

The position vector going from A to B is

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle 2, 4, -3 \rangle \text{ m}.\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(2)^2 + (4)^2 + (-3)^2} \text{ m} \\ &= \sqrt{29} \text{ m}.\end{aligned}$$

Divide $\mathbf{r}_{AB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{\langle 2, 4, -3 \rangle}{\sqrt{29}}$$

The force $\mathbf{F}_{AB}$ can now be written.

$$\mathbf{F}_{AB} = F_{AB}\hat{\mathbf{u}}_{AB} = 200\,\frac{\langle 2, 4, -3 \rangle}{\sqrt{29}} \text{ N} \approx \langle 74.3, 149, -111 \rangle \text{ N}$$

On the other hand, the position vector going from A to C is

$$\begin{aligned}\mathbf{r}_{AC} &= \mathbf{r}_C - \mathbf{r}_A \\ &= \langle -3, -4, -3 \rangle \text{ m}.\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{AC}| &= \sqrt{(-3)^2 + (-4)^2 + (-3)^2} \text{ m} \\ &= \sqrt{34} \text{ m}.\end{aligned}$$

Divide $\mathbf{r}_{AC}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \frac{\langle -3, -4, -3 \rangle}{\sqrt{34}}$$

The force $\mathbf{F}_{AC}$ can now be written.

$$\mathbf{F}_{AC} = F_{AC}\hat{\mathbf{u}}_{AC} = 150\,\frac{\langle -3, -4, -3 \rangle}{\sqrt{34}} \text{ N} \approx \langle -77.2, -103, -77.2 \rangle \text{ N}$$

Add the two forces to get the resultant.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_{AB} + \mathbf{F}_{AC} \\ &= 200\,\frac{\langle 2, 4, -3 \rangle}{\sqrt{29}} \text{ N} + 150\,\frac{\langle -3, -4, -3 \rangle}{\sqrt{34}} \text{ N} \\ &\approx \langle -2.90, 45.7, -189 \rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &\approx \sqrt{(-2.90)^2 + (45.7)^2 + (-189)} \text{ N} \\ &\approx 194 \text{ N}.\end{aligned}$$

Divide the resultant by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle -2.90, 45.7, -189 \rangle}{194}$$

The direction angles for the resultant can now be found.

$$\begin{cases} \cos\alpha \approx -\dfrac{2.90}{194} \\ \cos\beta \approx \dfrac{45.7}{194} \\ \cos\gamma \approx -\dfrac{189}{194} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 90.9^\circ \\ \beta \approx 76.4^\circ \\ \gamma \approx 166^\circ \end{cases}$$