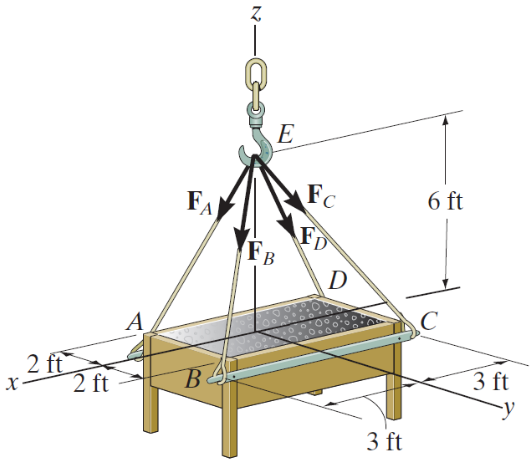
Problem 2-104

If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



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Solution

Write the position vectors to the points A, B, C, D, and E.

$$\mathbf{r}_A = \langle 3, -2, 0\rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 3, 2, 0\rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -3, 2, 0\rangle \text{ ft}$$

$$\mathbf{r}_D = \langle -3, -2, 0\rangle \text{ ft}$$

$$\mathbf{r}_E = \langle 0, 0, 6\rangle \text{ ft}$$

The position vector going from E to A is then

$$\begin{aligned}\mathbf{r}_{EA} &= \mathbf{r}_A - \mathbf{r}_E \\ &= \langle 3, -2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EA}| &= \sqrt{(3)^2 + (-2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EA}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EA} = \frac{\mathbf{r}_{EA}}{|\mathbf{r}_{EA}|} = \frac{\langle 3, -2, -6\rangle}{7}$$

The force $\mathbf{F}_A$ can now be written.

$$\mathbf{F}_A = F_A\hat{\mathbf{u}}_{EA} = 70\,\frac{\langle 3, -2, -6\rangle}{7} \text{ lb} = \langle 30, -20, -60\rangle \text{ lb}$$

On the other hand, the position vector going from E to B is

$$\begin{aligned}\mathbf{r}_{EB} &= \mathbf{r}_B - \mathbf{r}_E \\ &= \langle 3, 2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EB}| &= \sqrt{(3)^2 + (2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EB} = \frac{\mathbf{r}_{EB}}{|\mathbf{r}_{EB}|} = \frac{\langle 3, 2, -6\rangle}{7}$$

The force $\mathbf{F}_B$ can now be written.

$$\mathbf{F}_B = F_B\hat{\mathbf{u}}_{EB} = 70\,\frac{\langle 3, 2, -6\rangle}{7} \text{ lb} = \langle 30, 20, -60\rangle \text{ lb}$$

On the other hand, the position vector going from E to C is

$$\begin{aligned}\mathbf{r}_{EC} &= \mathbf{r}_C - \mathbf{r}_E \\ &= \langle -3, 2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EC}| &= \sqrt{(-3)^2 + (2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EC}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EC} = \frac{\mathbf{r}_{EC}}{|\mathbf{r}_{EC}|} = \frac{\langle -3, 2, -6\rangle}{7}$$

The force $\mathbf{F}_C$ can now be written.

$$\mathbf{F}_C = F_C\hat{\mathbf{u}}_{EC} = 70\,\frac{\langle -3, 2, -6\rangle}{7} \text{ lb} = \langle -30, 20, -60\rangle \text{ lb}$$

On the other hand, the position vector going from E to D is

$$\begin{aligned}\mathbf{r}_{ED} &= \mathbf{r}_D - \mathbf{r}_E \\ &= \langle -3, -2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{ED}| &= \sqrt{(-3)^2 + (-2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{ED}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{ED} = \frac{\mathbf{r}_{ED}}{|\mathbf{r}_{ED}|} = \frac{\langle -3, 2, -6 \rangle}{7}$$

The force $\mathbf{F}_D$ can now be written.

$$\mathbf{F}_D = F_D \hat{\mathbf{u}}_{ED} = 70\,\frac{\langle -3, -2, -6 \rangle}{7}\text{ lb} = \langle -30, -20, -60 \rangle\text{ lb}$$

Add the four forces to get their resultant.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D \\ &= \langle 30, -20, -60 \rangle\text{ lb} + \langle 30, 20, -60 \rangle\text{ lb} + \langle -30, 20, -60 \rangle\text{ lb} + \langle -30, -20, -60 \rangle\text{ lb} \\ &= \langle 0, 0, -240 \rangle\text{ lb}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(0)^2 + (0)^2 + (-240)^2}\text{ lb} \\ &= 240\text{ lb.}\end{aligned}$$

Divide the resultant by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} = \langle 0, 0, -1 \rangle$$

The direction angles for the resultant can now be found.

$$\begin{cases}\cos\alpha = 0 \\ \cos\beta = 0 \\ \cos\gamma = -1\end{cases} \quad \rightarrow \quad \begin{cases}\alpha \approx 90.0^\circ \\ \beta \approx 90.0^\circ \\ \gamma \approx 180^\circ\end{cases}$$