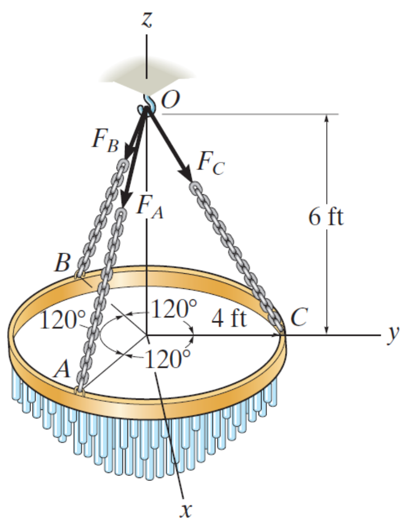
Problem 2-108

The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



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Solution

Write the position vectors to the points O, A, B, and C.

$$\mathbf{r}_O = \langle 0, 0, 6 \rangle \text{ ft}$$

$$\mathbf{r}_A = 4\langle \cos 30°, \cos 120°, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = 4\langle \cos 150°, \cos 120°, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 0, 4, 0 \rangle \text{ ft}$$

The position vector from O to A is then

$$\begin{aligned}\mathbf{r}_{OA} &= \mathbf{r}_A - \mathbf{r}_O \\ &= \langle 4\cos 30°, 4\cos 120°, -6 \rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{OA}| &= \sqrt{(4\cos 30°)^2 + (4\cos 120°)^2 + (-6)^2} \text{ ft} \\ &= 2\sqrt{13} \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{OA}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{OA} = \frac{\mathbf{r}_{OA}}{|\mathbf{r}_{OA}|} = \frac{\langle 4\cos 30°, 4\cos 120°, -6 \rangle}{2\sqrt{13}}$$

The force $\mathbf{F}_A$ can now be written.

$$\mathbf{F}_A = F_A \hat{\mathbf{u}}_{OA} = 60\,\frac{\langle 4\cos 30°, 4\cos 120°, -6\rangle}{2\sqrt{13}}\text{ lb} \approx \langle 28.8, -16.6, -49.9\rangle\text{ lb}$$

On the other hand, the position vector from O to B is

$$\begin{aligned}\mathbf{r}_{OB} &= \mathbf{r}_B - \mathbf{r}_O \\ &= \langle 4\cos 150°, 4\cos 120°, -6\rangle\text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{OB}| &= \sqrt{(4\cos 150°)^2 + (4\cos 120°)^2 + (-6)^2}\text{ ft} \\ &= 2\sqrt{13}\text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{OB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{OB} = \frac{\mathbf{r}_{OB}}{|\mathbf{r}_{OB}|} = \frac{\langle 4\cos 150°, 4\cos 120°, -6\rangle}{2\sqrt{13}}$$

The force $\mathbf{F}_B$ can now be written.

$$\mathbf{F}_B = F_B \hat{\mathbf{u}}_{OB} = 60\,\frac{\langle 4\cos 150°, 4\cos 120°, -6\rangle}{2\sqrt{13}}\text{ lb} \approx \langle -28.8, -16.6, -49.9\rangle\text{ lb}$$

On the other hand, the position vector from O to C is

$$\begin{aligned}\mathbf{r}_{OC} &= \mathbf{r}_C - \mathbf{r}_O \\ &= \langle 0, 4, -6\rangle\text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{OC}| &= \sqrt{(0)^2 + (4)^2 + (-6)^2}\text{ ft} \\ &= 2\sqrt{13}\text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{OC}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{OC} = \frac{\mathbf{r}_{OC}}{|\mathbf{r}_{OC}|} = \frac{\langle 0, 4, -6\rangle}{2\sqrt{13}}$$

The force $\mathbf{F}_C$ can now be written.

$$\mathbf{F}_C = F_C \hat{\mathbf{u}}_{OC} = 60\,\frac{\langle 0, 4, -6\rangle}{2\sqrt{13}}\text{ lb} \approx \langle 0, 33.3, -49.9\rangle\text{ lb}$$

Add these three forces to get the resultant.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C \\ &= 60\,\frac{\langle 4\cos 30°, 4\cos 120°, -6\rangle}{2\sqrt{13}}\text{ lb} + 60\,\frac{\langle 4\cos 150°, 4\cos 120°, -6\rangle}{2\sqrt{13}}\text{ lb} + 60\,\frac{\langle 0, 4, -6\rangle}{2\sqrt{13}}\text{ lb} \\ &= \left\langle 0, 0, -\frac{540}{\sqrt{13}}\right\rangle\text{ lb}\end{aligned}$$

Its magnitude is

$$
\begin{aligned}
|\mathbf{F}_R| &= \sqrt{(0)^2 + (0)^2 + \left(-\frac{540}{\sqrt{13}}\right)^2} \text{ lb} \\
&= \frac{540}{\sqrt{13}} \text{ lb} \\
&\approx 150. \text{ lb}.
\end{aligned}
$$

Divide $\mathbf{F}_R$ by its magnitude to get a unit vector in the same direction.

$$
\frac{\mathbf{F}_R}{|\mathbf{F}_R|} = \frac{\left\langle 0, 0, -\frac{540}{\sqrt{13}} \right\rangle}{\frac{540}{\sqrt{13}}} = \langle 0, 0, -1 \rangle
$$

The direction angles of the resultant can now be found.

$$
\begin{cases} \cos\alpha = 0 \\ \cos\beta = 0 \\ \cos\gamma = -1 \end{cases} \quad \rightarrow \quad \begin{cases} \alpha = 90.0^\circ \\ \beta = 90.0^\circ \\ \gamma = 180^\circ \end{cases}
$$