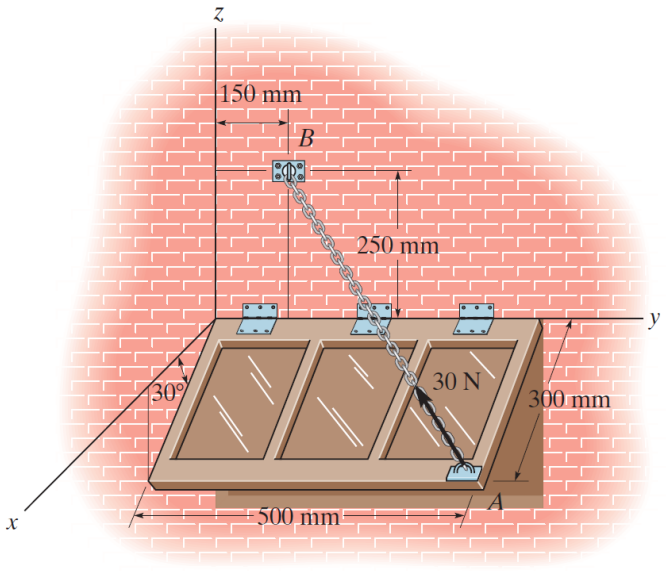
Problem 2-111

The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



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Solution

Write the position vectors to the points A and B.

$$\mathbf{r}_A = \langle 300\cos 30°, 500, -300\sin 30° \rangle \text{ mm}$$

$$\mathbf{r}_B = \langle 0, 150, 250 \rangle \text{ mm}$$

The position vector from A to B is then

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle -300\cos 30°, -350, 250 + 300\sin 30° \rangle \text{ mm}.\end{aligned}$$

Its magnitude is the length of the chain.

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(-300\cos 30°)^2 + (-350)^2 + (250 + 300\sin 30°)^2} \text{ mm} \\ &\approx 592 \text{ mm}\end{aligned}$$

Divide $\mathbf{r}_{AB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} \approx \frac{\langle -300\cos 30°, -350, 250 + 300\sin 30° \rangle}{592} \quad \rightarrow \quad \begin{cases} \cos\alpha \approx \dfrac{-300\cos 30°}{592} \\ \cos\beta \approx \dfrac{-350}{592} \\ \cos\gamma \approx \dfrac{250 + 300\sin 30°}{592} \end{cases}$$

The direction angles for the force are therefore

$$\begin{cases} \alpha \approx 116^\circ \\ \beta \approx 126^\circ \\ \gamma \approx 47.5^\circ \end{cases}.$$

Finally, write the force $\mathbf{F}$.

$$\mathbf{F} = F\hat{\mathbf{u}}_{AB} \approx 30\,\frac{\langle -300\cos 30^\circ, -350, 250 + 300\sin 30^\circ\rangle}{592}\ \text{N} \approx \langle -13.2, -17.7, 20.3\rangle\ \text{N}$$