Problem 2-112

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

Solution

Evaluate the given dot product.

$$\begin{aligned}
\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) &= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i A_i\right) \cdot \left[\left(\sum_{j=1}^{3} \boldsymbol{\delta}_j B_j\right) + \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j D_j\right)\right] \\
&= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i A_i\right) \cdot \left[\sum_{j=1}^{3} \boldsymbol{\delta}_j (B_j + D_j)\right] \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j) A_i (B_j + D_j) \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} A_i (B_j + D_j) \\
&= \sum_{i=1}^{3} A_i (B_i + D_i) \\
&= \sum_{i=1}^{3} (A_i B_i + A_i D_i) \\
&= \sum_{i=1}^{3} A_i B_i + \sum_{i=1}^{3} A_i D_i \\
&= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})
\end{aligned}$$