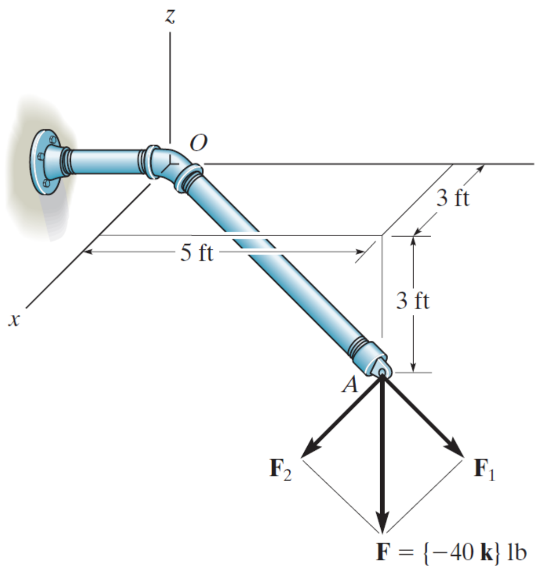
Problem 2-119

A force of $\mathbf{F} = \{-40\mathbf{k}\}$ lb acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.



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Solution

Write the position vectors to the points O and A.

$$\mathbf{r}_O = \langle 0, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_A = \langle 3, 5, -3 \rangle \text{ ft}$$

The unit vector in the direction from O to A is

$$\hat{\mathbf{u}}_{OA} = \frac{\mathbf{r}_A - \mathbf{r}_O}{|\mathbf{r}_A - \mathbf{r}_O|} = \frac{\langle 3, 5, -3 \rangle}{\sqrt{(3)^2 + (5)^2 + (-3)^2}} = \left\langle \frac{3}{\sqrt{43}}, \frac{5}{\sqrt{43}}, -\frac{3}{\sqrt{43}} \right\rangle.$$

Take the dot product of $\mathbf{F}$ with this unit vector to find the component of the force along the pipe's axis.

$$F_{\parallel} = \mathbf{F} \cdot \hat{\mathbf{u}}_{OA} = \langle 0, 0, -40 \rangle \cdot \left\langle \frac{3}{\sqrt{43}}, \frac{5}{\sqrt{43}}, -\frac{3}{\sqrt{43}} \right\rangle \text{ lb} = \frac{120}{\sqrt{43}} \text{ lb}$$

Therefore, the magnitude of the component parallel to the pipe's axis is

$$|F_{\parallel}| = \frac{120}{\sqrt{43}} \text{ lb} \approx 18.3 \text{ lb}.$$

Subtract the component of **F** parallel to the pipe's axis from **F** to get the component of **F** perpendicular to the pipe's axis.

$$\begin{aligned}
\mathbf{F}_\perp &= \mathbf{F} - \mathbf{F}_\parallel \\
&= \langle 0, 0, -40 \rangle \text{ lb} - |F_\parallel| \hat{\mathbf{u}}_{OA} \\
&= \langle 0, 0, -40 \rangle \text{ lb} - \frac{120}{\sqrt{43}} \left\langle \frac{3}{\sqrt{43}}, \frac{5}{\sqrt{43}}, -\frac{3}{\sqrt{43}} \right\rangle \text{ lb} \\
&= \left\langle -\frac{360}{43}, -\frac{600}{43}, \frac{360}{43} - 40 \right\rangle \text{ lb}
\end{aligned}$$

Therefore, the magnitude of the component perpendicular to the pipe's axis is

$$\begin{aligned}
|F_\perp| &= \sqrt{\left(-\frac{360}{43}\right)^2 + \left(-\frac{600}{43}\right)^2 + \left(\frac{360}{43} - 40\right)^2} \\
&\approx 35.6 \text{ lb.}
\end{aligned}$$