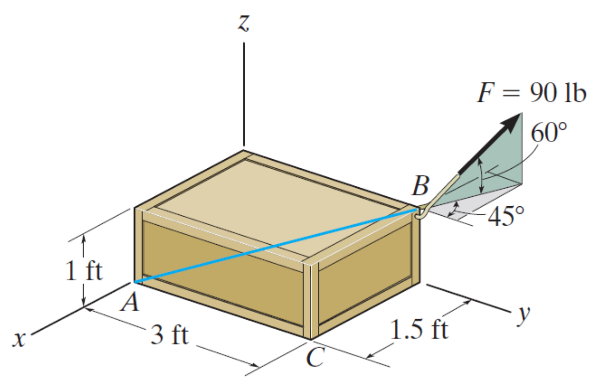
Problem 2-135

Determine the magnitudes of the components of the force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



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Solution

Write the position vectors to the points A and B.

$$\mathbf{r}_A = \langle 1.5, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 0, 3, 1 \rangle \text{ ft}$$

The unit vector going from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} = \frac{\langle -1.5, 3, 1 \rangle}{\sqrt{(-1.5)^2 + (3)^2 + (1)^2}}.$$

Write the force in component form.

$$\mathbf{F} = 90 \langle -\cos 60^\circ \sin 45^\circ, \cos 60^\circ \cos 45^\circ, \sin 60^\circ \rangle \text{ lb}$$

Take the dot product of $\mathbf{F}$ with $\hat{\mathbf{u}}_{AB}$ to get the component of the force along diagonal AB.

$$F_{\parallel} = \mathbf{F} \cdot \hat{\mathbf{u}}_{AB} = 90 \langle -\cos 60^\circ \sin 45^\circ, \cos 60^\circ \cos 45^\circ, \sin 60^\circ \rangle \cdot \frac{\langle -1.5, 3, 1 \rangle}{\sqrt{(-1.5)^2 + (3)^2 + (1)^2}} \text{ lb}$$

$$= \frac{45}{14}(9\sqrt{2} + 4\sqrt{3}) \text{ lb}$$

$$\approx 63.2 \text{ lb}$$

Therefore, the magnitude of the force's component along AB is

$$|F_{\parallel}| = \frac{45}{14}(9\sqrt{2} + 4\sqrt{3}) \text{ lb} \approx 63.2 \text{ lb}.$$

Subtract the component of **F** parallel to AB from **F** to get the component of **F** perpendicular to AB.

$$\begin{aligned}\mathbf{F}_\perp &= \mathbf{F} - \mathbf{F}_\parallel \\ &= 90\,\langle -\cos 60^\circ \sin 45^\circ, \cos 60^\circ \cos 45^\circ, \sin 60^\circ\rangle \text{ lb} - F_\parallel \hat{\mathbf{u}}_{AB} \\ &= 90\,\langle -\cos 60^\circ \sin 45^\circ, \cos 60^\circ \cos 45^\circ, \sin 60^\circ\rangle \text{ lb} - \frac{45}{14}(9\sqrt{2}+4\sqrt{3})\frac{\langle -1.5, 3, 1\rangle}{\sqrt{(-1.5)^2+(3)^2+(1)^2}} \text{ lb} \\ &= \left\langle -\frac{45}{49}(11\sqrt{2}-6\sqrt{3}), -\frac{45}{98}(5\sqrt{2}+24\sqrt{3}), -\frac{405}{49}(\sqrt{2}-5\sqrt{3})\right\rangle \text{ lb}\end{aligned}$$

Therefore, the magnitude of the force's component perpendicular to AB is

$$\begin{aligned}|F_\perp| &= \sqrt{\left[-\frac{45}{49}(11\sqrt{2}-6\sqrt{3})\right]^2 + \left[-\frac{45}{98}(5\sqrt{2}+24\sqrt{3})\right]^2 + \left[-\frac{405}{49}(\sqrt{2}-5\sqrt{3})\right]^2} \text{ lb} \\ &\approx 64.1 \text{ lb.}\end{aligned}$$