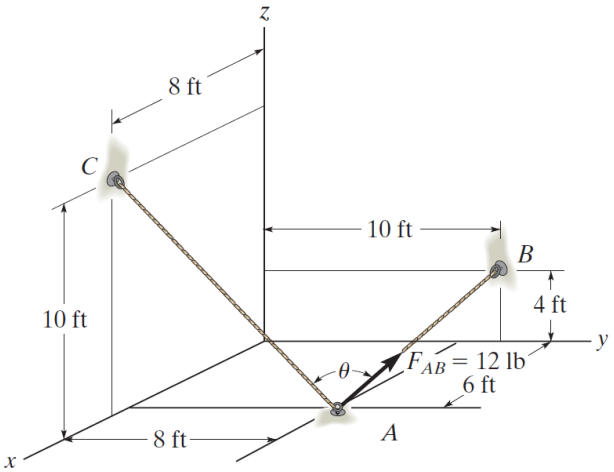
Problem 2-139

Determine the projected component of the force $F = 12$ lb acting in the direction of cable AC. Express the result as a Cartesian vector.



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Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle 6, 8, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 0, 10, 4 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 8, 0, 10 \rangle \text{ ft}$$

The unit vector going from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} = \frac{\langle -6, 2, 4 \rangle}{\sqrt{(-6)^2 + (2)^2 + (4)^2}},$$

and the unit vector going from A to C is

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} = \frac{\langle 2, -8, 10 \rangle}{\sqrt{(2)^2 + (-8)^2 + (10)^2}}.$$

Write the force acting along AB in component form.

$$\mathbf{F}_{AB} = F_{AB}\hat{\mathbf{u}}_{AB} = 12\,\frac{\langle -6, 2, 4 \rangle}{\sqrt{(-6)^2 + (2)^2 + (4)^2}} \text{ lb}$$

Take the dot product of $\mathbf{F}_{AB}$ with $\hat{\mathbf{u}}_{AC}$ to find the component of the force along cable AC.

$$F_{\parallel} = \mathbf{F}_{AB} \cdot \hat{\mathbf{u}}_{AC} = 12\,\frac{\langle -6, 2, 4 \rangle}{\sqrt{(-6)^2 + (2)^2 + (4)^2}} \cdot \frac{\langle 2, -8, 10 \rangle}{\sqrt{(2)^2 + (-8)^2 + (10)^2}} \text{ lb} = \frac{6\sqrt{3}}{7} \text{ lb} \approx 1.48 \text{ lb}$$

$$\mathbf{F}_{\parallel} = F_{\parallel}\hat{\mathbf{u}}_{AC} = \frac{6\sqrt{3}}{7}\,\frac{\langle 2, -8, 10 \rangle}{\sqrt{(2)^2 + (-8)^2 + (10)^2}} \text{ lb} \approx \{0.229, -0.916, 1.15\} \text{ lb}$$