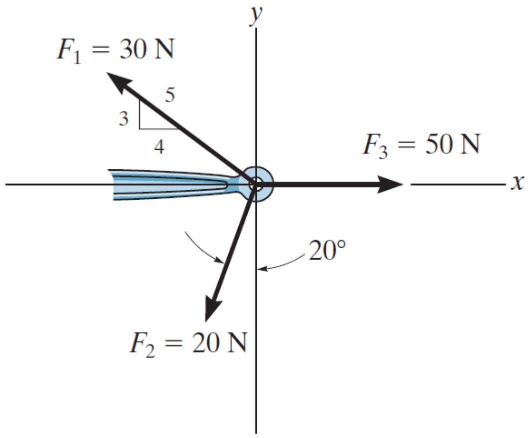
Problem 2-18

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



Probs. 2–17/18

Solution

Begin by finding α, the angle that $\mathbf{F}_1$ is above the negative x-axis.

$$\tan\alpha = \frac{3}{4} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9^\circ$$

Write down the vectors in component form.

$$\mathbf{F}_1 = 30\langle -\cos\alpha, \sin\alpha\rangle \text{ N}$$

$$\mathbf{F}_2 = 20\langle -\sin 20^\circ, -\cos 20^\circ\rangle \text{ N}$$

$$\mathbf{F}_3 = 50\langle 1, 0\rangle \text{ N}$$

Add $\mathbf{F}_2$ and $\mathbf{F}_3$ to get $\mathbf{F}'$.

$$\begin{aligned} \mathbf{F}' &= \mathbf{F}_2 + \mathbf{F}_3 \\ &= \langle -20\sin 20^\circ + 50, -20\cos 20^\circ\rangle \text{ N} \end{aligned}$$

Then add $\mathbf{F}'$ and $\mathbf{F}_1$ to get $\mathbf{F}_R$.

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}' + \mathbf{F}_1 \\ &= \langle -20\sin 20^\circ + 50 - 30\cos\alpha, -20\cos 20^\circ + 30\sin\alpha\rangle \text{ N} \\ &\approx \langle 19.2, -0.794\rangle \text{ N} \end{aligned}$$

Its magnitude is

$$|\mathbf{F}_R| = \sqrt{(-20\sin 20^\circ + 50 - 30\cos\alpha)^2 + (-20\cos 20^\circ + 30\sin\alpha)^2} \approx 19.2 \text{ N},$$

and the direction it points in counterclockwise from the positive x-axis is

$$\tan\phi = \frac{-20\cos 20^\circ + 30\sin\alpha}{-20\sin 20^\circ + 50 - 30\cos\alpha} \quad \rightarrow \quad \phi \approx -2.37^\circ.$$