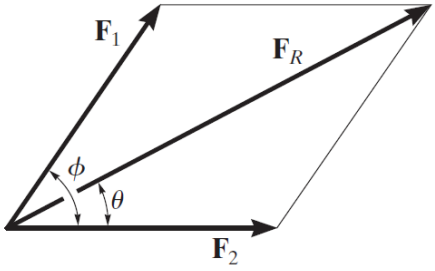
Problem 2-45

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.



Prob. 2–45

Solution

Write each of the forces in component form, assuming that $\mathbf{F}_2$ points in the positive x-direction and that the y-axis points straight up as usual.

$$\mathbf{F}_1 = F_1 \langle \cos\phi, \sin\phi \rangle$$

$$\mathbf{F}_2 = F_2 \langle 1, 0 \rangle$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \langle F_1 \cos\phi + F_2, F_1 \sin\phi \rangle\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(F_1\cos\phi + F_2)^2 + (F_1\sin\phi)^2} \\ &= \sqrt{F_1^2\cos^2\phi + 2F_1F_2\cos\phi + F_2^2 + F_1^2\sin^2\phi} \\ &= \sqrt{F_1^2(\cos^2\phi + \sin^2\phi) + F_2^2 + 2F_1F_2\cos\phi} \\ &= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi},\end{aligned}$$

and the direction it points in counterclockwise from the positive x-axis is

$$\tan\theta = \frac{F_1\sin\phi}{F_1\cos\phi + F_2} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_1\cos\phi + F_2}\right).$$