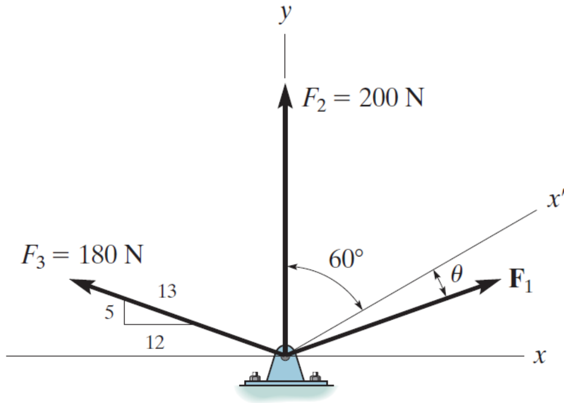
Problem 2-48

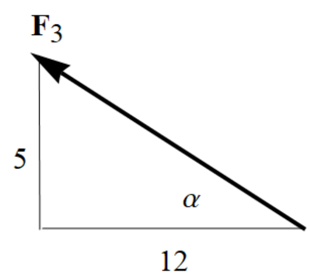
Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.



Probs. 2–48/49

Solution

Begin by finding the angle α that $\mathbf{F}_3$ makes with the x-axis.



$$\tan\alpha = \frac{5}{12} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.6^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = F_1 \langle \sin(\theta + 60^\circ), \cos(\theta + 60^\circ) \rangle \text{ N}$$

$$\mathbf{F}_2 = 200 \langle 0, 1 \rangle \text{ N}$$

$$\mathbf{F}_3 = 180 \langle -\cos\alpha, \sin\alpha \rangle \text{ N} = 180 \left\langle -\frac{12}{13}, \frac{5}{13} \right\rangle \text{ N} = \left\langle -\frac{2160}{13}, \frac{900}{13} \right\rangle \text{ N}$$

$$\mathbf{F}_R = 800 \langle \sin 60^\circ, \cos 60^\circ \rangle \text{ N}$$

Add $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ together to get the resultant force $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\langle 800 \sin 60^\circ, 800 \cos 60^\circ \rangle = \left\langle F_1 \sin(\theta + 60^\circ) - \frac{2160}{13}, F_1 \cos(\theta + 60^\circ) + 200 + \frac{900}{13} \right\rangle \text{ N}$$

Match the components to get a system of equations.

$$800 \sin 60^\circ = F_1 \sin(\theta + 60^\circ) - \frac{2160}{13}$$

$$800 \cos 60^\circ = F_1 \cos(\theta + 60^\circ) + 200 + \frac{900}{13}$$

Solve this second equation for F_1

$$F_1 = \frac{800 \cos 60^\circ - 200 - \frac{900}{13}}{\cos(\theta + 60^\circ)}$$

and substitute it into the first equation.

$$800 \sin 60^\circ = \left[\frac{800 \cos 60^\circ - 200 - \frac{900}{13}}{\cos(\theta + 60^\circ)}\right] \sin(\theta + 60^\circ) - \frac{2160}{13}$$

Solve for θ.

$$800 \sin 60^\circ + \frac{2160}{13} = \left(800 \cos 60^\circ - 200 - \frac{900}{13}\right) \tan(\theta + 60^\circ)$$

$$\tan(\theta + 60^\circ) = \frac{800 \sin 60^\circ + \frac{2160}{13}}{800 \cos 60^\circ - 200 - \frac{900}{13}}$$

$$\theta + 60^\circ \approx 81.3^\circ$$

$$\theta \approx 21.3^\circ$$

Plug this value of θ into the formula for F_1.

$$F_1 = \frac{800 \cos 60^\circ - 200 - \frac{900}{13}}{\cos(\theta + 60^\circ)} \approx 869 \text{ N}$$