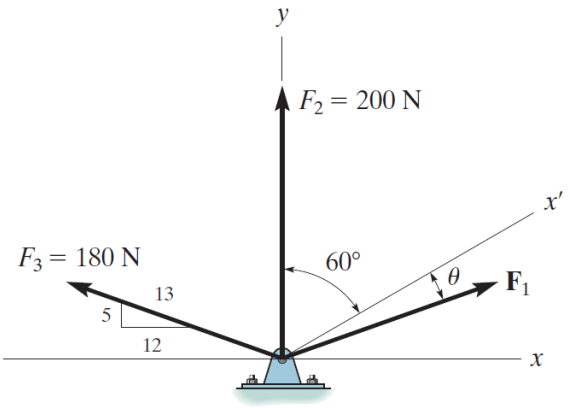
Problem 2-49

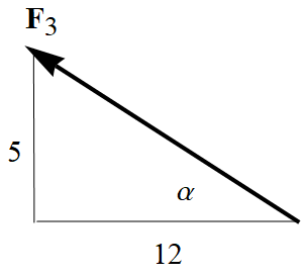
If $F_1 = 300$ N and $\theta = 10°$, determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force acting on the bracket.



Probs. 2–48/49

Solution

Begin by finding the angle α that $\mathbf{F}_3$ makes with the x-axis.



$$\tan\alpha = \frac{5}{12} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.6°$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 300\,\langle \sin 70°, \cos 70° \rangle \text{ N}$$

$$\mathbf{F}_2 = 200\,\langle 0, 1 \rangle \text{ N}$$

$$\mathbf{F}_3 = 180\,\langle -\cos\alpha, \sin\alpha \rangle \text{ N} = 180\left\langle -\frac{12}{13}, \frac{5}{13} \right\rangle \text{ N} = \left\langle -\frac{2160}{13}, \frac{900}{13} \right\rangle \text{ N}$$

Add $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ together to get the resultant force $\mathbf{F}_R$.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= \left\langle 300\sin 70° - \frac{2160}{13}, 300\cos 70° + 200 + \frac{900}{13} \right\rangle \text{ N} \\ &\approx \langle 116, 372 \rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{\left(300\sin 70^\circ - \frac{2160}{13}\right)^2 + \left(300\cos 70^\circ + 200 + \frac{900}{13}\right)^2} \\ &\approx 389 \text{ N},\end{aligned}$$

and the direction it points in counterclockwise from the positive x-axis is

$$\tan\phi = \frac{300\cos 70^\circ + 200 + \frac{900}{13}}{300\sin 70^\circ - \frac{2160}{13}} \quad \rightarrow \quad \phi \approx 72.7^\circ.$$

Measuring the counterclockwise angle from the positive x'-axis instead gives $\phi - 30^\circ \approx 42.7^\circ$.