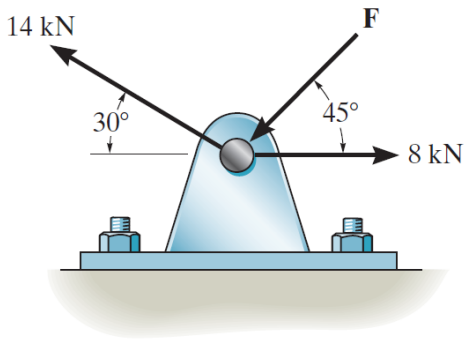
Problem 2-55

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



Prob. 2–55

Solution

Write each of the forces in component form.

$$\mathbf{F}_1 = 14\,\langle -\cos 30°, \sin 30°\rangle \text{ kN}$$

$$\mathbf{F}_2 = 8\,\langle 1, 0\rangle \text{ kN}$$

$$\mathbf{F} = F\,\langle -\cos 45°, -\sin 45°\rangle \text{ kN}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F} \\ &= \langle -14\cos 30° + 8 - F\cos 45°, 14\sin 30° - F\sin 45°\rangle \text{ kN}\end{aligned}$$

Its magnitude is

$$|\mathbf{F}_R| = F_R = \sqrt{(-14\cos 30° + 8 - F\cos 45°)^2 + (14\sin 30° - F\sin 45°)^2}.$$

In order to minimize the magnitude, take the derivative of F_R (or rather F_R^2 since it has no square root)

$$\begin{aligned}\frac{d}{dF}(F_R^2) &= \frac{d}{dF}\left[(-14\cos 30° + 8 - F\cos 45°)^2 + (14\sin 30° - F\sin 45°)^2\right] \\ &= 2(-14\cos 30° + 8 - F\cos 45°)\cdot\frac{d}{dF}(-14\cos 30° + 8 - F\cos 45°) \\ &\qquad + 2(14\sin 30° - F\sin 45°)\cdot\frac{d}{dF}(14\sin 30° - F\sin 45°) \\ &= 2(-14\cos 30° + 8 - F\cos 45°)\cdot(-\cos 45°) \\ &\qquad + 2(14\sin 30° - F\sin 45°)\cdot(-\sin 45°) \\ &= 2F + 7\sqrt{6} - 15\sqrt{2}\end{aligned}$$

and set it equal to zero.

$$2F + 7\sqrt{6} - 15\sqrt{2} = 0$$

Solve for F.

$$F = \frac{15\sqrt{2} - 7\sqrt{6}}{2} \approx 2.03 \text{ kN}$$

This is the magnitude of **F** that produces the smallest resultant force. Plug it into the formula for F_R to find this minimal resultant force.

$$F_R = \sqrt{\left[-14\cos 30^\circ + 8 - \left(\frac{15\sqrt{2} - 7\sqrt{6}}{2}\right)\cos 45^\circ\right]^2 + \left[14\sin 30^\circ - \left(\frac{15\sqrt{2} - 7\sqrt{6}}{2}\right)\sin 45^\circ\right]^2}$$

$$\approx 7.87 \text{ kN}$$

The graph of F_R versus F below confirms these numbers.