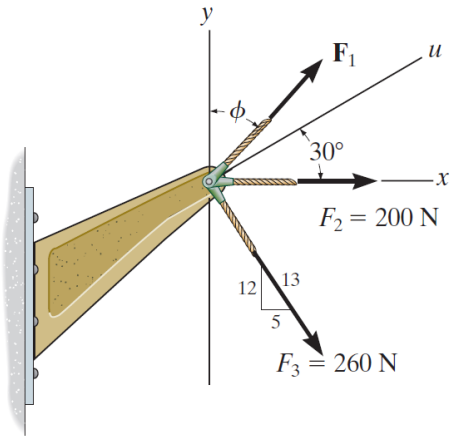
Problem 2-56

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



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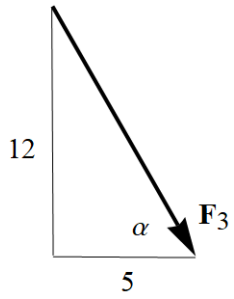
Solution

Begin by finding the angle that $\mathbf{F}_3$ makes with the x-axis.



$$\tan\alpha = \frac{12}{5} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{12}{5}\right) \approx 67.4^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = F_1 \langle \sin\phi, \cos\phi \rangle \text{ N}$$

$$\mathbf{F}_2 = 200 \langle 1, 0 \rangle \text{ N}$$

$$\mathbf{F}_3 = 260 \langle \cos\alpha, -\sin\alpha \rangle \text{ N} = 260 \left\langle \frac{5}{13}, -\frac{12}{13} \right\rangle \text{ N} = \langle 100, -240 \rangle \text{ N}$$

$$\mathbf{F}_R = 450 \langle \cos 30^\circ, \sin 30^\circ \rangle \text{ N}$$

Add them together to get the resultant force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\langle 450\cos 30^\circ, 450\sin 30^\circ \rangle = \langle F_1 \sin\phi + 200 + 100, F_1 \cos\phi - 240 \rangle \text{ N}$$

Match the components to get a system of equations.

$$450\cos 30^\circ = F_1 \sin\phi + 200 + 100$$

$$450\sin 30^\circ = F_1 \cos\phi - 240$$

Solve this second equation for F_1

$$F_1 = \frac{450\sin 30^\circ + 240}{\cos\phi}$$

and substitute it into the first equation.

$$450\cos 30^\circ = \left(\frac{450\sin 30^\circ + 240}{\cos\phi}\right)\sin\phi + 300$$

$$450\cos 30^\circ = (450\sin 30^\circ + 240)\tan\phi + 300$$

$$\tan\phi = \frac{450\cos 30^\circ - 300}{450\sin 30^\circ + 240}$$

$$\phi \approx 10.9^\circ$$

Plug this value for ϕ into the formula for F_1.

$$F_1 = \frac{450\sin 30^\circ + 240}{\cos\phi} \approx 474 \text{ N}$$