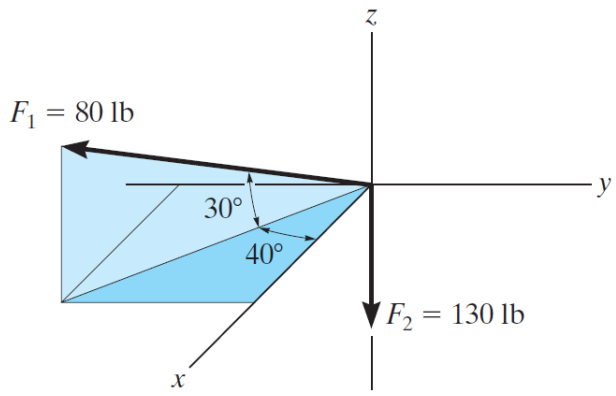
Problem 2-63

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



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Solution

Write each of the forces in component form.

$$\mathbf{F}_1 = 80\,\langle \cos 30^\circ \cos 40^\circ, -\cos 30^\circ \sin 40^\circ, \sin 30^\circ \rangle \text{ lb}$$

$$\mathbf{F}_2 = 130\,\langle 0, 0, -1 \rangle \text{ lb}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \langle 80\cos 30^\circ \cos 40^\circ, -80\cos 30^\circ \sin 40^\circ, 80\sin 30^\circ - 130 \rangle \text{ lb} \\ &\approx \langle 53.1, -44.5, -90 \rangle \text{ lb}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(80\cos 30^\circ \cos 40^\circ)^2 + (-80\cos 30^\circ \sin 40^\circ)^2 + (80\sin 30^\circ - 130)^2} \\ &\approx 114 \text{ lb}.\end{aligned}$$

A unit vector in the direction of the resultant force is

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} = \frac{\langle 80\cos 30^\circ \cos 40^\circ, -80\cos 30^\circ \sin 40^\circ, 80\sin 30^\circ - 130 \rangle}{\sqrt{(80\cos 30^\circ \cos 40^\circ)^2 + (-80\cos 30^\circ \sin 40^\circ)^2 + (80\sin 30^\circ - 130)^2}} \approx \frac{\langle 53.1, -44.5, -90 \rangle}{114}.$$

Therefore, the direction angles are

$$\begin{cases} \cos\alpha \approx \dfrac{53.1}{114} \\ \cos\beta \approx -\dfrac{44.5}{114} \\ \cos\gamma \approx -\dfrac{90}{114} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 62.1^\circ \\ \beta \approx 113^\circ \\ \gamma \approx 142^\circ \end{cases}.$$

The resultant force is illustrated below with the direction angles.