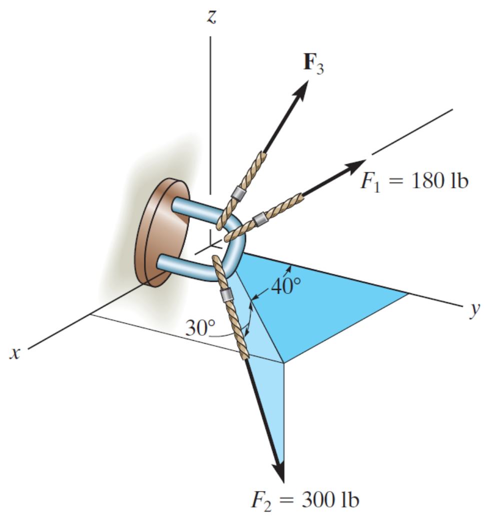
Problem 2-68

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



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Solution

Write each of the forces in component form.

$$\mathbf{F}_1 = 180 \langle -1, 0, 0 \rangle \text{ lb}$$

$$\mathbf{F}_2 = 300 \langle \cos 30^\circ \sin 40^\circ, \cos 30^\circ \cos 40^\circ, -\sin 30^\circ \rangle \text{ lb}$$

$$\mathbf{F}_3 = F_3 \langle \cos \alpha_3, \cos \beta_3, \cos \gamma_3 \rangle \text{ lb}$$

$$\mathbf{F}_R = \langle 0, 0, 0 \rangle \text{ lb}$$

Add them together to get the resultant force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\langle 0, 0, 0 \rangle \text{ lb} = \langle -180 + 300 \cos 30^\circ \sin 40^\circ + F_3 \cos \alpha_3, 300 \cos 30^\circ \cos 40^\circ + F_3 \cos \beta_3, -300 \sin 30^\circ + F_3 \cos \gamma_3 \rangle \text{ lb}$$

Match the components to obtain a system of equations.

$$0 = -180 + 300 \cos 30^\circ \sin 40^\circ + F_3 \cos \alpha_3$$

$$0 = 300 \cos 30^\circ \cos 40^\circ + F_3 \cos \beta_3$$

$$0 = -300 \sin 30^\circ + F_3 \cos \gamma_3$$

Solve for the terms with F_3.

$$F_3 \cos \alpha_3 = 180 - 300 \cos 30^\circ \sin 40^\circ \quad (1)$$

$$F_3 \cos \beta_3 = -300 \cos 30^\circ \cos 40^\circ \quad (2)$$

$$F_3 \cos \gamma_3 = 300 \sin 30^\circ \quad (3)$$

Square both sides of each equation and add the respective sides together to get F_3.

$$F_3^2(\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3) = (180 - 300\cos 30^\circ \sin 40^\circ)^2 + (-300\cos 30^\circ \cos 40^\circ)^2 + (300\sin 30^\circ)^2$$

$$F_3^2(1) = (180 - 300\cos 30^\circ \sin 40^\circ)^2 + (-300\cos 30^\circ \cos 40^\circ)^2 + (300\sin 30^\circ)$$

$$F_3 = \sqrt{(180 - 300\cos 30^\circ \sin 40^\circ)^2 + (-300\cos 30^\circ \cos 40^\circ)^2 + (300\sin 30^\circ)}$$

$$F_3 \approx 250.\ \text{lb}$$

Plug this into equations (1), (2), and (3) to determine α_3, β_3, and γ_3.

$$\begin{cases} \cos\alpha_3 = \dfrac{180 - 300\cos 30^\circ \sin 40^\circ}{F_3} \\ \cos\beta_3 = \dfrac{-300\cos 30^\circ \cos 40^\circ}{F_3} \\ \cos\gamma_3 = \dfrac{300\sin 30^\circ}{F_3} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha_3 \approx 87.0^\circ \\ \beta_3 \approx 143^\circ \\ \gamma_3 \approx 53.1^\circ \end{cases}$$