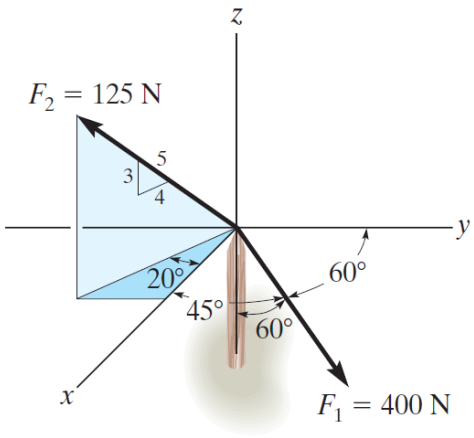
Problem 2-69

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



Prob. 2–69

Solution

Let θ be the angle that $\mathbf{F}_2$ is above the xy-plane.

$$\tan\theta = \frac{3}{4} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 400\,\langle \cos 45^\circ, \cos 60^\circ, -\cos 60^\circ \rangle \text{ N}$$

$$\mathbf{F}_2 = 125\,\langle \cos\theta \cos 20^\circ, -\cos\theta \sin 20^\circ, \sin\theta \rangle \text{ N} = 125\left\langle \frac{4}{5}\cos 20^\circ, -\frac{4}{5}\sin 20^\circ, \frac{3}{5} \right\rangle \text{ N}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \langle 400\cos 45^\circ + 100\cos 20^\circ, 400\cos 60^\circ - 100\sin 20^\circ, -400\cos 60^\circ + 75 \rangle \text{ N} \\ &\approx \langle 377, 166, -125 \rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(400\cos 45^\circ + 100\cos 20^\circ)^2 + (400\cos 60^\circ - 100\sin 20^\circ)^2 + (-400\cos 60^\circ + 75)^2} \text{ N} \\ &\approx 430. \text{ N}\end{aligned}$$

Divide the resultant force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 377, 166, -125 \rangle \text{ N}}{430. \text{ N}}$$

The direction angles for $\mathbf{F}_R$ can now be determined.

$$\begin{cases} \cos\alpha \approx \dfrac{377}{430} \\ \cos\beta \approx \dfrac{166}{430} \\ \cos\gamma \approx -\dfrac{125}{430} \end{cases} \rightarrow \begin{cases} \alpha \approx 28.9^\circ \\ \beta \approx 67.3^\circ \\ \gamma \approx 107^\circ \end{cases}$$

The resultant force is illustrated below in the coordinate system.