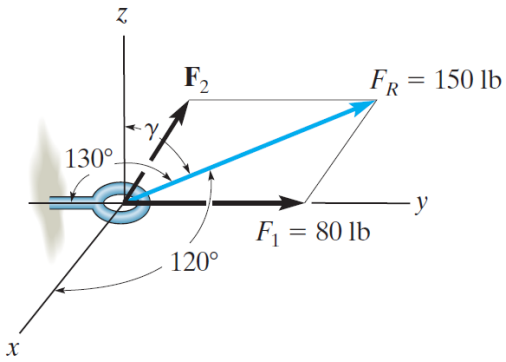
Problem 2-72

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



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Solution

Begin by finding γ, the angle that $\mathbf{F}_R$ makes with the positive z-axis.

$$\cos^2 120^\circ + \cos^2 130^\circ + \cos^2 \gamma = 1 \quad \rightarrow \quad \gamma = \cos^{-1}\left(\sqrt{1 - \cos^2 120^\circ - \cos^2 130^\circ}\right) \approx 54.5^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 80\,\langle 0, 1, 0\rangle \text{ lb}$$

$$\mathbf{F}_2 = F_2\,\langle \cos\alpha_2, \cos\beta_2, \cos\gamma_2\rangle \text{ lb}$$

$$\mathbf{F}_R = 150\,\langle \cos 120^\circ, -\cos 130^\circ, \cos\gamma\rangle \text{ lb}$$

Add them together to get the resultant force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\langle 150\cos 120^\circ, -150\cos 130^\circ, 150\cos\gamma\rangle \text{ lb} = \langle F_2\cos\alpha_2, 80 + F_2\cos\beta_2, F_2\cos\gamma_2\rangle \text{ lb}$$

Match the components to get a system of equations.

$$150\cos 120^\circ = F_2\cos\alpha_2$$

$$-150\cos 130^\circ = 80 + F_2\cos\beta_2$$

$$150\cos\gamma = F_2\cos\gamma_2$$

Solve for the terms with F_2.

$$F_2\cos\alpha_2 = 150\cos 120^\circ \tag{1}$$

$$F_2\cos\beta_2 = -80 - 150\cos 130^\circ \tag{2}$$

$$F_2\cos\gamma_2 = 150\cos\gamma \tag{3}$$

Square both sides of each equation and add the respective sides together to get F_2.

$$F_2^2(\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2) = (150\cos 120°)^2 + (-80 - 150\cos 130°)^2 + (150\cos\gamma)^2$$

$$F_2^2(1) = (150\cos 120°)^2 + (-80 - 150\cos 130°)^2 + (150\cos\gamma)^2$$

$$F_2 = \sqrt{(150\cos 120°)^2 + (-80 - 150\cos 130°)^2 + (150\cos\gamma)^2}$$

$$F_2 \approx 116 \text{ lb}$$

Plug this into equations (1), (2), and (3) to determine α_2, β_2, and γ_2.

$$\begin{cases} \cos\alpha_2 = \dfrac{150\cos 120°}{F_2} \\ \cos\beta_2 = \dfrac{-80 - 150\cos 130°}{F_2} \\ \cos\gamma_2 = \dfrac{150\cos\gamma}{F_2} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha_2 \approx 130° \\ \beta_2 \approx 81.9° \\ \gamma_2 \approx 41.4° \end{cases}$$