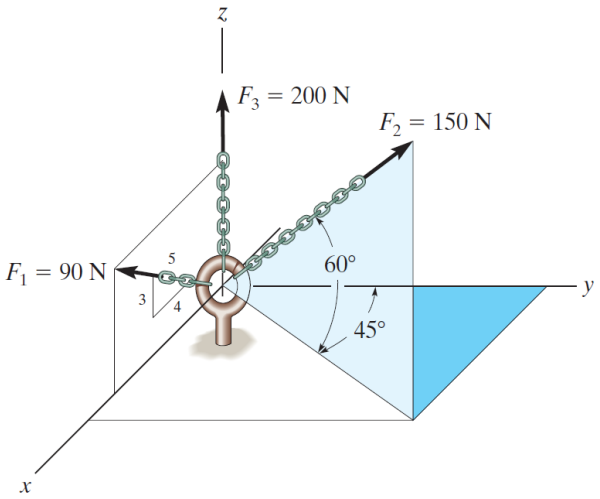
Problem 2-74

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



Probs. 2–73/74

Solution

Let θ be the angle that $\mathbf{F}_1$ makes with the x-axis.

$$\tan\theta = \frac{3}{4} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 90\,\langle \cos\theta, 0, \sin\theta \rangle \text{ N} = 90\left\langle \frac{4}{5}, 0, \frac{3}{5} \right\rangle \text{ N} = \langle 72, 0, 54 \rangle \text{ N}$$

$$\mathbf{F}_2 = 150\,\langle \cos 60^\circ \sin 45^\circ, \cos 60^\circ \cos 45^\circ, \sin 60^\circ \rangle \text{ N}$$

$$\mathbf{F}_3 = 200\,\langle 0, 0, 1 \rangle \text{ N}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= \langle 72 + 150\cos 60^\circ \sin 45^\circ, 150\cos 60^\circ \cos 45^\circ, 54 + 150\sin 60^\circ + 200 \rangle \text{ N} \\ &\approx \langle 125, 53.0, 384 \rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(72 + 150\cos 60^\circ \sin 45^\circ)^2 + (150\cos 60^\circ \cos 45^\circ)^2 + (54 + 150\sin 60^\circ + 200)^2} \text{ N} \\ &\approx 407 \text{ N}.\end{aligned}$$

Divide the resultant force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 125, 53.0, 384 \rangle \text{ N}}{407 \text{ N}}$$

The direction angles for the resultant force can now be found.

$$\begin{cases} \cos\alpha \approx \dfrac{125}{407} \\ \cos\beta \approx \dfrac{53.0}{407} \\ \cos\gamma \approx \dfrac{384}{407} \end{cases} \rightarrow \begin{cases} \alpha \approx 72.1^\circ \\ \beta \approx 82.5^\circ \\ \gamma \approx 19.5^\circ \end{cases}$$

Below is an illustration of the resultant force and its direction angles.