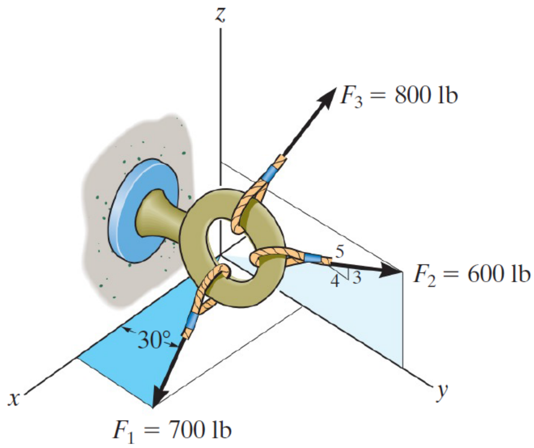
Problem 2-81

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 60°$ and $\gamma_3 = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



Probs. 2–81/82/83

Solution

Let θ be the angle that $\mathbf{F}_2$ makes with the y-axis.

$$\tan\theta = \frac{3}{4} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 700\,\langle \cos 30°, \sin 30°, 0\rangle \text{ lb}$$

$$\mathbf{F}_2 = 600\,\langle 0, \cos\theta, \sin\theta\rangle \text{ lb} = 600\left\langle 0, \frac{4}{5}, \frac{3}{5}\right\rangle \text{ lb} = \langle 0, 480, 360\rangle \text{ lb}$$

$$\mathbf{F}_3 = 800\,\langle \cos 120°, \cos 60°, \cos 45°\rangle \text{ lb}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= \langle 700\cos 30° + 800\cos 120°, 700\sin 30° + 480 + 800\cos 60°, 360 + 800\cos 45°\rangle \text{ lb} \\ &\approx \langle 206, 1230, 926\rangle \text{ lb}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(700\cos 30° + 800\cos 120°)^2 + (700\sin 30° + 480 + 800\cos 60°)^2 + (360 + 800\cos 45°)^2} \text{ lb} \\ &\approx 1.55 \times 10^3 \text{ lb}.\end{aligned}$$

Divide the resultant force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 206, 1230, 926\rangle \text{ lb}}{1.55 \times 10^3 \text{ lb}}$$

The direction angles for the resultant force can now be found.

$$\begin{cases} \cos\alpha \approx \dfrac{206}{1.55\times 10^3} \\ \cos\beta \approx \dfrac{1230}{1.55\times 10^3} \\ \cos\gamma \approx \dfrac{926}{1.55\times 10^3} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 82.4^\circ \\ \beta \approx 37.6^\circ \\ \gamma \approx 53.4^\circ \end{cases}$$