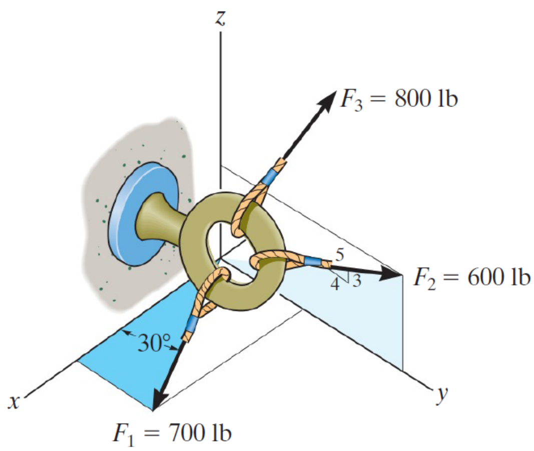
Problem 2-83

If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.



Probs. 2–81/82/83

Solution

Let θ be the angle that $\mathbf{F}_2$ makes with the y-axis.

$$\tan\theta = \frac{3}{4} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 700\,\langle \cos 30°, \sin 30°, 0\rangle \text{ lb}$$

$$\mathbf{F}_2 = 600\,\langle 0, \cos\theta, \sin\theta\rangle \text{ lb} = 600\left\langle 0, \frac{4}{5}, \frac{3}{5}\right\rangle \text{ lb} = \langle 0, 480, 360\rangle \text{ lb}$$

$$\mathbf{F}_3 = 800\,\langle \cos\alpha_3, \cos\beta_3, \cos\gamma_3\rangle \text{ lb}$$

$$\mathbf{F}_R = F_R\,\langle 0, \cos 30°, \sin 30°\rangle \text{ lb}$$

Add them together to get the resultant force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\langle 0, F_R\cos 30°, F_R\sin 30°\rangle \text{ lb} = \langle 700\cos 30° + 800\cos\alpha_3, 700\sin 30° + 480 + 800\cos\beta_3, 360 + 800\cos\gamma_3\rangle \text{ lb}$$

Match the components to get a system of equations.

$$0 = 700\cos 30° + 800\cos\alpha_3$$

$$F_R\cos 30° = 700\sin 30° + 480 + 800\cos\beta_3$$

$$F_R\sin 30° = 360 + 800\cos\gamma_3$$

Solve for the direction angles.

$$\cos\alpha_3 = -\frac{7\cos 30^\circ}{8}$$
$$\cos\beta_3 = \frac{F_R\cos 30^\circ - 700\sin 30^\circ - 480}{800}$$
$$\cos\gamma_3 = \frac{F_R\sin 30^\circ - 360}{800}$$

Square both sides of each equation and add the respective sides to get F_R.

$$\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3 = \left(-\frac{7\cos 30^\circ}{8}\right)^2 + \left(\frac{F_R\cos 30^\circ - 700\sin 30^\circ - 480}{800}\right)^2 + \left(\frac{F_R\sin 30^\circ - 360}{800}\right)^2$$

$$1 = \left(-\frac{7\cos 30^\circ}{8}\right)^2 + \left(\frac{F_R\cos 30^\circ - 700\sin 30^\circ - 480}{800}\right)^2 + \left(\frac{F_R\sin 30^\circ - 360}{800}\right)^2$$

$$1 = \frac{F_R^2}{640\,000} - \frac{83\sqrt{3}F_R}{64\,000} - \frac{9F_R}{16\,000} + \frac{593}{320}$$

$$\frac{F_R^2}{640\,000} - \frac{83\sqrt{3}F_R}{64\,000} - \frac{9F_R}{16\,000} + \frac{273}{320} = 0$$

$$F_R^2 - (830\sqrt{3} + 360)F_R + 546\,000 = 0$$

$$F_R = \frac{(830\sqrt{3} + 360) \pm \sqrt{(830\sqrt{3} + 360)^2 - 4(546\,000)}}{2}$$

$$F_R \approx \{387, 1.41 \times 10^3\} \text{ lb}$$

There are two possible magnitudes for the resultant force. If $F_R \approx 387$ lb, then

$$\begin{cases} \cos\alpha_3 = -\dfrac{7\cos 30^\circ}{8} \\ \cos\beta_3 = \dfrac{F_R\cos 30^\circ - 700\sin 30^\circ - 480}{800} \\ \cos\gamma_3 = \dfrac{F_R\sin 30^\circ - 360}{800} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha_3 \approx 139^\circ \\ \beta_3 \approx 128^\circ \\ \gamma_3 \approx 102^\circ \end{cases}.$$

And if $F_R \approx 1.41 \times 10^3$ lb, then

$$\begin{cases} \cos\alpha_3 = -\dfrac{7\cos 30^\circ}{8} \\ \cos\beta_3 = \dfrac{F_R\cos 30^\circ - 700\sin 30^\circ - 480}{800} \\ \cos\gamma_3 = \dfrac{F_R\sin 30^\circ - 360}{800} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha_3 \approx 139^\circ \\ \beta_3 \approx 60.7^\circ \\ \gamma_3 \approx 64.4^\circ \end{cases}.$$