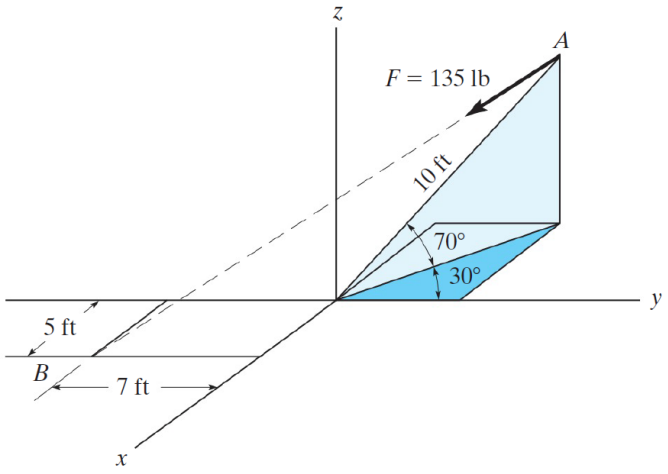
Problem 2-87

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



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Solution

Write the position vectors to points A and B in component form.

$$\mathbf{r}_A = 10\,\langle -\cos 70^\circ \sin 30^\circ, \cos 70^\circ \cos 30^\circ, \sin 70^\circ \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 5, -7, 0 \rangle \text{ ft}$$

The position vector from A to B is then

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle 5 + 10\cos 70^\circ \sin 30^\circ, -7 - 10\cos 70^\circ \cos 30^\circ, -10\sin 70^\circ \rangle \text{ ft} \\ &\approx \langle 6.71, -9.96, -9.40 \rangle \text{ ft},\end{aligned}$$

and its magnitude is

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(5 + 10\cos 70^\circ \sin 30^\circ)^2 + (-7 - 10\cos 70^\circ \cos 30^\circ)^2 + (-10\sin 70^\circ)^2} \text{ ft} \\ &\approx 15.3 \text{ ft}.\end{aligned}$$

A unit vector in the direction from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} \approx \frac{\langle 6.71, -9.96, -9.40 \rangle}{15.3} \quad \Rightarrow \quad \begin{cases} \cos\alpha \approx \dfrac{6.71}{15.3} \\ \cos\beta \approx -\dfrac{9.96}{15.3} \\ \cos\gamma \approx -\dfrac{9.40}{15.3} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 63.9^\circ \\ \beta \approx 131^\circ \\ \gamma \approx 128^\circ \end{cases}.$$

Therefore, the force acting from A to B is

$$\mathbf{F} = F\hat{\mathbf{u}}_{AB} \approx 135\,\frac{\langle 6.71, -9.96, -9.40 \rangle}{15.3} \text{ lb} \approx \langle 59.4, -88.2, -83.2 \rangle \text{ lb}.$$