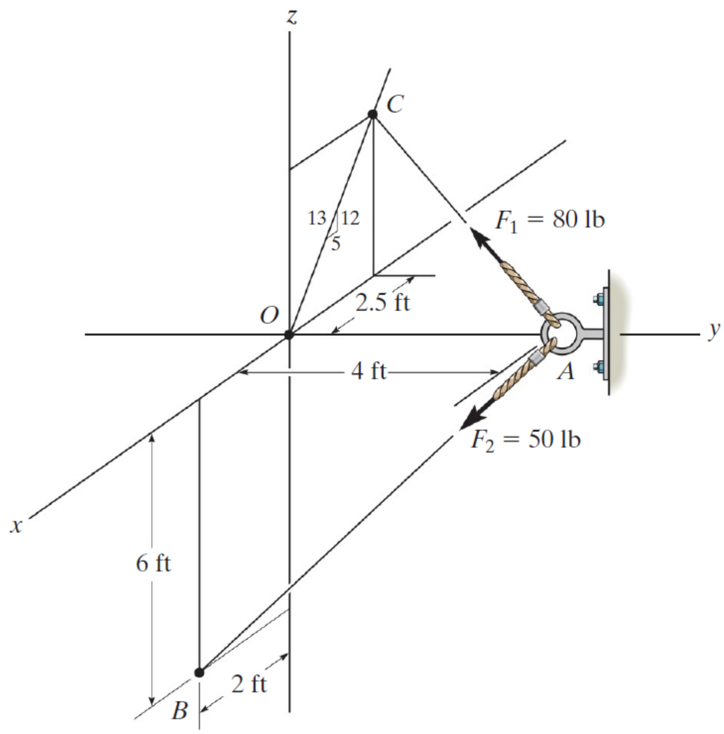
# Problem 2-88

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.




Prob. 2–88

---

**Solution**

Write the position vectors to points $A$, $B$, and $C$ in component form.

$$\mathbf{r}_A = \langle 0, 4, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 2, 0, -6 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -2.5, 0, 6 \rangle \text{ ft}$$

The position vector from $A$ to $B$ is then

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle 2 - 0, 0 - 4, -6 - 0 \rangle \text{ ft} \\ &= \langle 2, -4, -6 \rangle \text{ ft},\end{aligned}$$

and its magnitude is

$$|\mathbf{r}_{AB}| = \sqrt{(2)^2 + (-4)^2 + (-6)^2} \text{ ft} = 2\sqrt{14} \text{ ft}.$$

A unit vector in the direction from $A$ to $B$ is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{\langle 2, -4, -6 \rangle}{2\sqrt{14}},$$

so the force acting from $A$ to $B$ is

$$\mathbf{F}_2 = F_2\hat{\mathbf{u}}_{AB} = 50\,\frac{\langle 2, -4, -6\rangle}{2\sqrt{14}}\text{ lb} \approx \langle 13.4, -26.7, -40.1\rangle\text{ lb}.$$

On the other hand, the position vector from $A$ to $C$ is

$$\begin{aligned}\mathbf{r}_{AC} &= \mathbf{r}_C - \mathbf{r}_A \\ &= \langle -2.5 - 0, 0 - 4, 6 - 0\rangle\text{ ft} \\ &= \langle -2.5, -4, 6\rangle\text{ ft},\end{aligned}$$

and its magnitude is

$$|\mathbf{r}_{AC}| = \sqrt{(-2.5)^2 + (-4)^2 + (6)^2}\text{ ft} = \frac{\sqrt{233}}{2}\text{ ft}.$$

A unit vector in the direction from $A$ to $C$ is

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \frac{\langle -2.5, -4, 6\rangle}{\frac{\sqrt{233}}{2}},$$

so the force acting from $A$ to $C$ is

$$\mathbf{F}_1 = F_1\hat{\mathbf{u}}_{AC} = 80\,\frac{\langle -2.5, -4, 6\rangle}{\frac{\sqrt{233}}{2}}\text{ lb} \approx \langle -26.2, -41.9, 62.9\rangle\text{ lb}.$$

Add the two forces to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= 80\,\frac{\langle -2.5, -4, 6\rangle}{\frac{\sqrt{233}}{2}}\text{ lb} + 50\,\frac{\langle 2, -4, -6\rangle}{2\sqrt{14}}\text{ lb} \\ &= \left\langle -\frac{400}{\sqrt{233}} + \frac{50}{\sqrt{14}}, \frac{-640}{\sqrt{233}} - \frac{100}{\sqrt{14}}, \frac{960}{\sqrt{233}} - \frac{150}{\sqrt{14}}\right\rangle\text{ lb} \\ &\approx \langle -12.8, -68.7, 22.8\rangle\text{ lb}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &\approx \sqrt{(-12.8)^2 + (-68.7)^2 + (22.8)^2}\text{ lb} \\ &\approx 73.5\text{ lb}.\end{aligned}$$

Divide the resultant force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle -12.8, -68.7, 22.8\rangle}{73.5}$$

The direction angles for the resultant can now be found.

$$\begin{cases}\cos\alpha \approx -\dfrac{12.8}{73.5} \\ \cos\beta \approx -\dfrac{68.7}{73.5} \\ \cos\gamma \approx \dfrac{22.8}{73.5}\end{cases} \quad\rightarrow\quad \begin{cases}\alpha \approx 100^\circ \\ \beta \approx 159^\circ \\ \gamma \approx 71.9^\circ\end{cases}$$