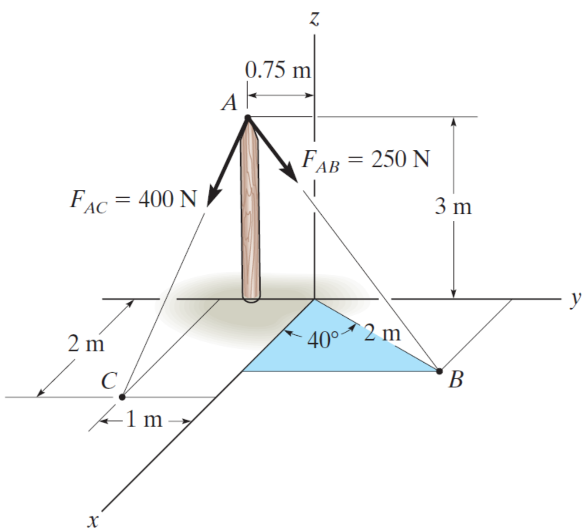
Problem 2-92

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–92

Solution

Write the position vectors to points A, B, and C in component form.

$$\mathbf{r}_A = \langle 0, -0.75, 3\rangle \text{ m}$$

$$\mathbf{r}_B = 2\langle \cos 40^\circ, \sin 40^\circ, 0\rangle \text{ m}$$

$$\mathbf{r}_C = \langle 2, -1, 0\rangle \text{ m}$$

The position vector from A to B is then

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle 2\cos 40^\circ, 2\sin 40^\circ + 0.75, -3\rangle \text{ m.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(2\cos 40^\circ)^2 + (2\sin 40^\circ + 0.75)^2 + (-3)^2} \text{ m} \\ &\approx 3.94 \text{ m.}\end{aligned}$$

Divide $\mathbf{r}_{AB}$ by its magnitude to get a unit vector pointing from A to B.

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{\langle 2\cos 40^\circ, 2\sin 40^\circ + 0.75, -3\rangle}{\sqrt{(2\cos 40^\circ)^2 + (2\sin 40^\circ + 0.75)^2 + (-3)^2}}$$

The force along line AB can now be written.

$$\mathbf{F}_{AB} = F_{AB}\hat{\mathbf{u}}_{AB} = 250\,\frac{\langle 2\cos 40^\circ, 2\sin 40^\circ + 0.75, -3\rangle}{\sqrt{(2\cos 40^\circ)^2 + (2\sin 40^\circ + 0.75)^2 + (-3)^2}} \text{ N} \approx \langle 97.3, 129, -191\rangle \text{ N}$$

On the other hand, the position vector from A to C is

$$\begin{aligned}\mathbf{r}_{AC} &= \mathbf{r}_C - \mathbf{r}_A \\ &= \langle 2, -0.25, -3 \rangle \text{ m.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{AC}| &= \sqrt{(2)^2 + (-0.25)^2 + (-3)^2} \text{ m} \\ &\approx 3.61 \text{ m.}\end{aligned}$$

Divide $\mathbf{r}_{AC}$ by its magnitude to get a unit vector pointing from A to C.

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \frac{\langle 2, -0.25, -3 \rangle}{\sqrt{(2)^2 + (-0.25)^2 + (-3)^2}}$$

The force along line AC can now be written.

$$\mathbf{F}_{AC} = F_{AC}\hat{\mathbf{u}}_{AC} = 400 \frac{\langle 2, -0.25, -3 \rangle}{\sqrt{(2)^2 + (-0.25)^2 + (-3)^2}} \text{ N} \approx \langle 221, -27.7, -332 \rangle \text{ N}$$

Add the two forces to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_{AB} + \mathbf{F}_{AC} \\ &= 250 \frac{\langle 2\cos 40^\circ, 2\sin 40^\circ + 0.75, -3 \rangle}{\sqrt{(2\cos 40^\circ)^2 + (2\sin 40^\circ + 0.75)^2 + (-3)^2}} \text{ N} + 400 \frac{\langle 2, -0.25, -3 \rangle}{\sqrt{(2)^2 + (-0.25)^2 + (-3)^2}} \text{ N} \\ &\approx \langle 319, 102, -523 \rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &\approx \sqrt{(319)^2 + (102)^2 + (-523)^2} \text{ N} \\ &\approx 620. \text{ N.}\end{aligned}$$

Divide the resultant by its magnitude to get a unit vector with the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 319, 102, -523 \rangle}{620.}$$

The direction angles of the resultant can now be found.

$$\begin{cases} \cos\alpha \approx \dfrac{319}{620} \\ \cos\beta \approx \dfrac{102}{620} \\ \cos\gamma \approx -\dfrac{523}{620} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 59.1^\circ \\ \beta \approx 80.6^\circ \\ \gamma \approx 147^\circ \end{cases}$$