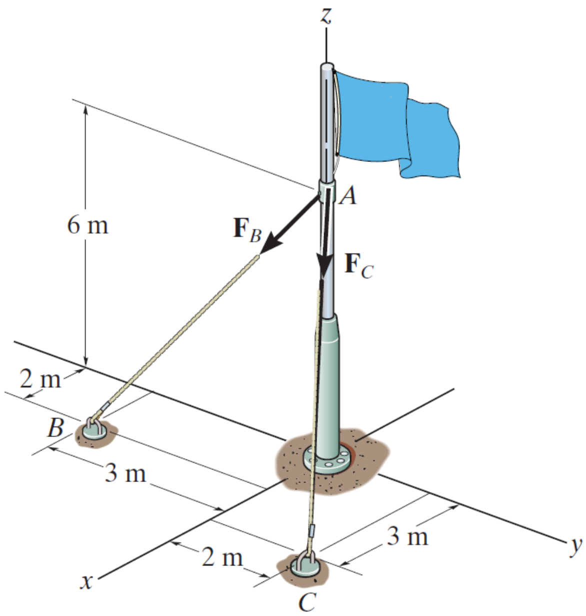
Problem 2-94

If $F_B = 700$ N and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



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Solution

Write the position vectors to points A, B, and C in component form.

$$\mathbf{r}_A = \langle 0, 0, 6 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 2, -3, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle 3, 2, 0 \rangle \text{ m}$$

The position vector from A to B is then

$$\begin{aligned} \mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle 2, -3, -6 \rangle \text{ m.} \end{aligned}$$

Its magnitude is

$$\begin{aligned} |\mathbf{r}_{AB}| &= \sqrt{(2)^2 + (-3)^2 + (-6)^2} \text{ m} \\ &= 7 \text{ m.} \end{aligned}$$

Divide $\mathbf{r}_{AB}$ by its magnitude to get a unit vector pointing from A to B.

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{\langle 2, -3, -6 \rangle}{7}$$

The force along line AB can now be written.

$$\mathbf{F}_B = F_B\hat{\mathbf{u}}_{AB} = 700\,\frac{\langle 2, -3, -6\rangle}{7}\text{ N} = \langle 200, -300, -600\rangle\text{ N}$$

On the other hand, the position vector from A to C is

$$\begin{aligned}\mathbf{r}_{AC} &= \mathbf{r}_C - \mathbf{r}_A\\ &= \langle 3, 2, -6\rangle\text{ m.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{AC}| &= \sqrt{(3)^2 + (2)^2 + (-6)^2}\text{ m}\\ &= 7\text{ m.}\end{aligned}$$

Divide $\mathbf{r}_{AC}$ by its magnitude to get a unit vector pointing from A to C.

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \frac{\langle 3, 2, -6\rangle}{7}$$

The force along line AC can now be written.

$$\mathbf{F}_C = F_C\hat{\mathbf{u}}_{AC} = 560\,\frac{\langle 3, 2, -6\rangle}{7}\text{ N} = \langle 240, 160, -480\rangle\text{ N}$$

Add the two forces to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C\\ &= \langle 200, -300, -600\rangle\text{ N} + \langle 240, 160, -480\rangle\text{ N}\\ &= \langle 440, -140, -1080\rangle\text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(440)^2 + (-140)^2 + (-1080)^2}\text{ N}\\ &\approx 1.17\times 10^3\text{ N.}\end{aligned}$$

Divide the resultant by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 440, -140, -1080\rangle}{1.17\times 10^3}$$

The direction angles of the resultant can now be found.

$$\begin{cases}\cos\alpha \approx \dfrac{440}{1.17\times 10^3}\\[2ex] \cos\beta \approx -\dfrac{140}{1.17\times 10^3}\\[2ex] \cos\gamma \approx -\dfrac{1080}{1.17\times 10^3}\end{cases} \quad\rightarrow\quad \begin{cases}\alpha \approx 68.0^\circ\\ \beta \approx 96.8^\circ\\ \gamma \approx 157^\circ\end{cases}$$