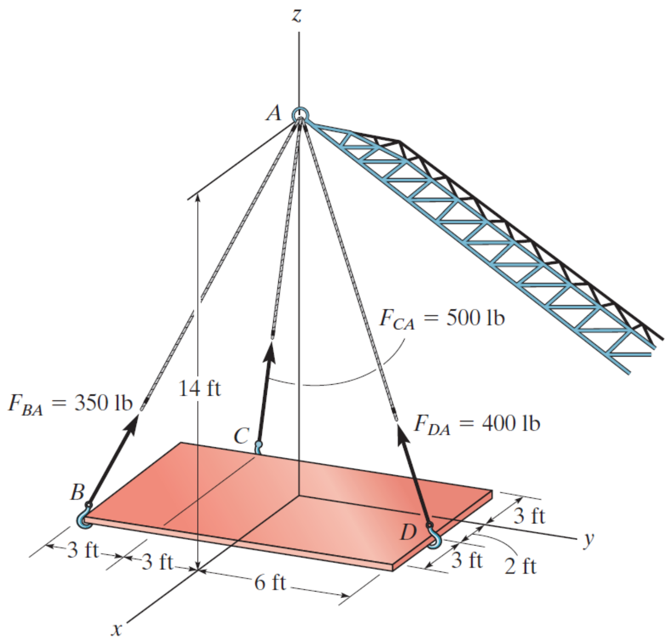
Problem 2-95

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



Prob. 2–95

Solution

Write the position vectors to points A, B, C, and D in component form.

$$\mathbf{r}_A = \langle 0, 0, 14 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 5, -6, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -3, -3, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 2, 6, 0 \rangle \text{ ft}$$

The position vector from B to A is then

$$\begin{aligned}\mathbf{r}_{BA} &= \mathbf{r}_A - \mathbf{r}_B \\ &= \langle -5, 6, 14 \rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{BA}| &= \sqrt{(-5)^2 + (6)^2 + (14)^2} \text{ ft} \\ &= \sqrt{257} \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{BA}$ by its magnitude to get a unit vector pointing from B to A.

$$\hat{\mathbf{u}}_{BA} = \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = \frac{\langle -5, 6, 14 \rangle}{\sqrt{257}}$$

The force along line BA can now be written.

$$\mathbf{F}_{BA} = F_{BA}\hat{\mathbf{u}}_{BA} = 350\,\frac{\langle -5, 6, 14\rangle}{\sqrt{257}}\text{ lb} \approx \langle -109, 131, 306\rangle\text{ lb}$$

On the other hand, the position vector from C to A is

$$\begin{aligned}\mathbf{r}_{CA} &= \mathbf{r}_A - \mathbf{r}_C \\ &= \langle 3, 3, 14\rangle\text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{CA}| &= \sqrt{(3)^2 + (3)^2 + (14)^2}\text{ ft} \\ &= \sqrt{214}\text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{CA}$ by its magnitude to get a unit vector pointing from C to A.

$$\hat{\mathbf{u}}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = \frac{\langle 3, 3, 14\rangle}{\sqrt{214}}$$

The force along line CA can now be written.

$$\mathbf{F}_{CA} = F_{CA}\hat{\mathbf{u}}_{CA} = 500\,\frac{\langle 3, 3, 14\rangle}{\sqrt{214}}\text{ lb} \approx \langle 103, 103, 479\rangle\text{ lb}$$

On the other hand, the position vector from D to A is

$$\begin{aligned}\mathbf{r}_{DA} &= \mathbf{r}_A - \mathbf{r}_D \\ &= \langle -2, -6, 14\rangle\text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{DA}| &= \sqrt{(-2)^2 + (-6)^2 + (14)^2}\text{ ft} \\ &= 2\sqrt{59}\text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{DA}$ by its magnitude to get a unit vector pointing from D to A.

$$\hat{\mathbf{u}}_{DA} = \frac{\mathbf{r}_{DA}}{|\mathbf{r}_{DA}|} = \frac{\langle -2, -6, 14\rangle}{2\sqrt{59}}$$

The force along line DA can now be written.

$$\mathbf{F}_{DA} = F_{DA}\hat{\mathbf{u}}_{DA} = 400\,\frac{\langle -2, -6, 14\rangle}{2\sqrt{59}}\text{ lb} \approx \langle -52.1, -156, 365\rangle\text{ lb}$$