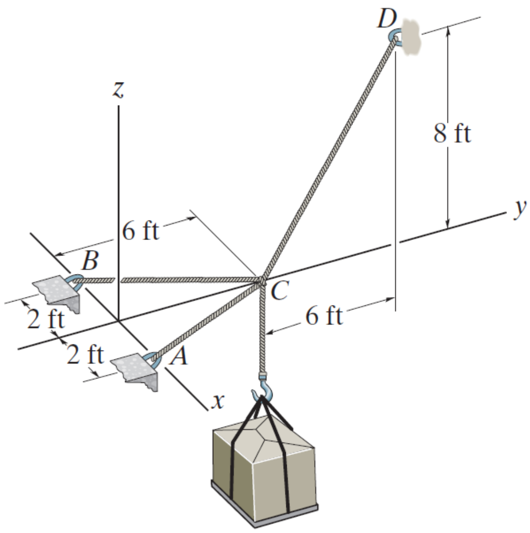
Problem R3-7

Determine the force in each cable needed to support the 500-lb load.



Prob. R3–7

Solution

Write position vectors to points A, B, C and D.

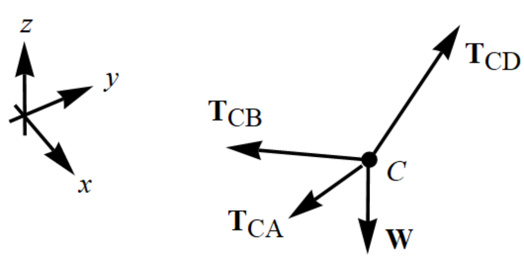
$$\mathbf{r}_A = \langle 2, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle -2, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 0, 6, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 0, 12, 8 \rangle \text{ ft}$$

Draw a free-body diagram for the knot at C.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{CA}} + \mathbf{T}_{\text{CB}} + \mathbf{T}_{\text{CD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{CA}}\hat{\mathbf{u}}_{\text{CA}} + T_{\text{CB}}\hat{\mathbf{u}}_{\text{CB}} + T_{\text{CD}}\hat{\mathbf{u}}_{\text{CD}} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{\text{CA}}\frac{\mathbf{r}_A - \mathbf{r}_C}{|\mathbf{r}_A - \mathbf{r}_C|} + T_{\text{CB}}\frac{\mathbf{r}_B - \mathbf{r}_C}{|\mathbf{r}_B - \mathbf{r}_C|} + T_{\text{CD}}\frac{\mathbf{r}_D - \mathbf{r}_C}{|\mathbf{r}_D - \mathbf{r}_C|} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{\text{CA}}\frac{\langle 2-0,0-6,0-0\rangle}{\sqrt{(2-0)^2+(0-6)^2+(0-0)^2}}+T_{\text{CB}}\frac{\langle -2-0,0-6,0-0\rangle}{\sqrt{(-2-0)^2+(0-6)^2+(0-0)^2}}$$
$$+T_{\text{CD}}\frac{\langle 0-0,12-6,8-0\rangle}{\sqrt{(0-0)^2+(12-6)^2+(8-0)^2}}+W\langle 0,0,-1\rangle=\mathbf{0}$$

$$T_{\text{CA}}\left\langle \frac{1}{\sqrt{10}},-\frac{3}{\sqrt{10}},0\right\rangle+T_{\text{CB}}\left\langle -\frac{1}{\sqrt{10}},-\frac{3}{\sqrt{10}},0\right\rangle+T_{\text{CD}}\left\langle 0,\frac{3}{5},\frac{4}{5}\right\rangle+W\langle 0,0,-1\rangle=\mathbf{0}$$

$$\left\langle \frac{1}{\sqrt{10}}T_{\text{CA}}-\frac{1}{\sqrt{10}}T_{\text{CB}},\ -\frac{3}{\sqrt{10}}T_{\text{CA}}-\frac{3}{\sqrt{10}}T_{\text{CB}}+\frac{3}{5}T_{\text{CD}},\ \frac{4}{5}T_{\text{CD}}-W\right\rangle=\langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned}\frac{1}{\sqrt{10}}T_{\text{CA}}-\frac{1}{\sqrt{10}}T_{\text{CB}}&=0\\ -\frac{3}{\sqrt{10}}T_{\text{CA}}-\frac{3}{\sqrt{10}}T_{\text{CB}}+\frac{3}{5}T_{\text{CD}}&=0\\ \frac{4}{5}T_{\text{CD}}-W&=0\end{aligned}\right\}$$

Solve it for T_{CA}, T_{CB}, and T_{CD} and then plug in $W = 500$ lb.

$$T_{\text{CA}}=\frac{1}{4}\sqrt{\frac{5}{2}}W\approx 198\text{ lb}$$

$$T_{\text{CB}}=\frac{1}{4}\sqrt{\frac{5}{2}}W\approx 198\text{ lb}$$

$$T_{\text{CD}}=\frac{5}{4}W=625\text{ lb}$$