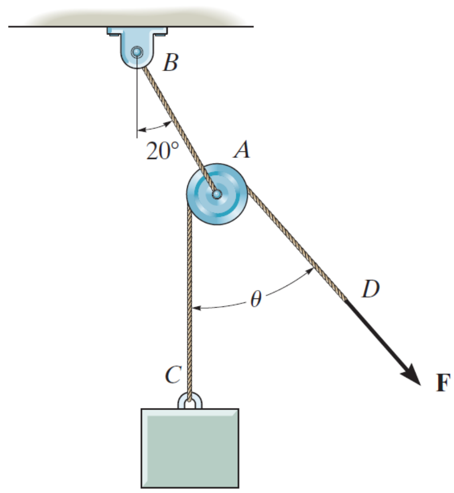
Problem 3-11

Determine the maximum weight W of the block that can be suspended in the position shown if cords AB and CAD can each support a maximum tension of 80 lb. Also, what is the angle θ for equilibrium?



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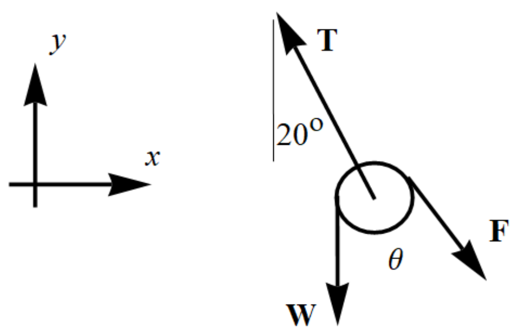
Solution

Draw a free-body diagram for the pulley.



In order for the pulley to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0 : \qquad F \sin\theta - T \sin 20^\circ = 0$$

$$\sum F_y = 0 : \qquad T \cos 20^\circ - F \cos\theta - W = 0$$

Because the pulley is frictionless, the tension in cord CAD is the same everywhere: $F = W$. And since cord AB supports everything, it has the highest tension. Set $T = 80$ lb.

$$W \sin\theta - 80 \sin 20^\circ = 0 \tag{1}$$

$$80 \cos 20^\circ - W \cos\theta - W = 0 \tag{2}$$

Solve equation (1) for W

$$W = \frac{80 \sin 20^\circ}{\sin \theta}$$

and substitute it into equation (2).

$$80 \cos 20^\circ - \frac{80 \sin 20^\circ}{\sin \theta}(\cos \theta + 1) = 0$$

$$\cot 20^\circ - \frac{\cos \theta + 1}{\sin \theta} = 0$$

$$\frac{\cos \theta + 1}{\sin \theta} = \cot 20^\circ$$

$$\frac{\cos^2 \theta + 2 \cos \theta + 1}{\sin^2 \theta} = \cot^2 20^\circ$$

$$\frac{\cos^2 \theta + 2 \cos \theta + 1}{1 - \cos^2 \theta} = \cot^2 20^\circ$$

$$\cos^2 \theta + 2 \cos \theta + 1 = \cot^2 20^\circ - \cot^2 20^\circ \cos^2 \theta$$

$$(1 + \cot^2 20^\circ) \cos^2 \theta + 2 \cos \theta + (1 - \cot^2 20^\circ) = 0$$

$$(\csc^2 20^\circ) \cos^2 \theta + 2 \cos \theta + (1 - \cot^2 20^\circ) = 0$$

$$\cos \theta = \frac{-2 \pm \sqrt{4 - 4(\csc^2 20^\circ)(1 - \cot^2 20^\circ)}}{2(\csc^2 20^\circ)}$$

$$\cos \theta \approx \{-1, 0.766\}$$

$$\theta = \{180^\circ, 40^\circ\}$$

Since $0^\circ < \theta < 90^\circ$, we choose $\theta = 40^\circ$. Plug this value into the formula for W to determine the maximum weight.

$$W = \frac{80 \sin 20^\circ}{\sin \theta} \approx 42.6 \text{ lb}$$