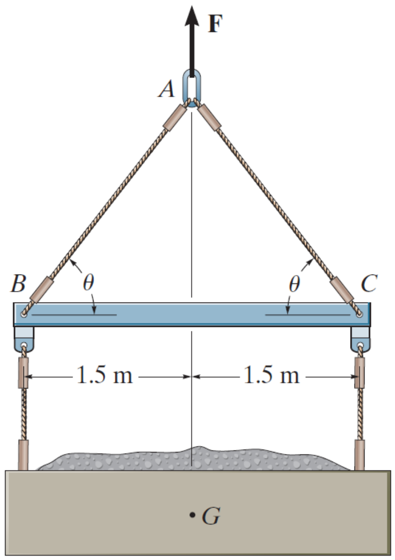
Problem 3-12

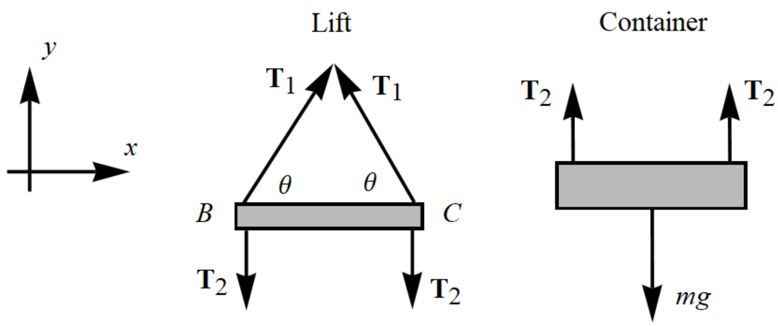
The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ. If the maximum tension allowed in each cable is 5 kN, determine the shortest length of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.



Prob. 3–12

Solution

Because the center of gravity is located at G, there is symmetry and the tension will be the same in both pairs of cables. Draw one free-body diagram for the lift and one free-body diagram for the container.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T_1\cos\theta - T_1\cos\theta = 0 \qquad 0 = 0$$

$$\sum F_y = 0: \qquad 2T_1\sin\theta - 2T_2 = 0 \qquad 2T_2 - mg = 0$$

Only the equations in the y-direction are relevant here.

Solve for $2T_2$

$$2T_2 = mg$$

and plug it into the equation involving T_1.

$$2T_1 \sin\theta - (mg) = 0$$

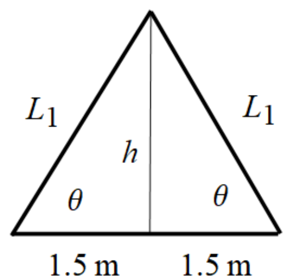
Solve for T_1, the tension in cables AB and AC.

$$T_1(\theta) = \frac{mg}{2\sin\theta} = \frac{(500\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{2\sin\theta} \approx \frac{2453}{\sin\theta}\text{ N}$$

Suppose this tension can't be higher than 5000 N.

$$\sin\theta = \frac{mg}{2(5000)}$$

Draw the triangle formed by the lift and cables in order to rewrite the left side.



Therefore, the shortest possible length of cables AB and AC is

$$\sin\theta = \frac{h}{L_1} = \frac{\sqrt{L_1^2 - 1.5^2}}{L_1} = \frac{mg}{2(5000)}$$

$$\frac{L_1^2 - 1.5^2}{L_1^2} = \frac{m^2 g^2}{10\,000^2}$$

$$1 - \frac{1.5^2}{L_1^2} = \frac{m^2 g^2}{10\,000^2}$$

$$\frac{1.5^2}{L_1^2} = 1 - \frac{m^2 g^2}{10\,000^2}$$

$$L_1 = \sqrt{\frac{1.5^2}{1 - \frac{m^2 g^2}{10\,000^2}}}$$

$$L_1 \approx 1.72\text{ m}.$$